PUNA INTERNATIONAL SCHOOL

- **CLASS – 10**
- **SUBJECT - MATHS**
- **CHAPTER - 6**

SAMPLE NOTE-BOOK

Chapter – 6 - Triangles

Exercise 6.1

= **Fill in the blanks using the correct word given in brackets:**

(i) All circles are ________________. (congruent, similar) (ii) All

squares are ________________. (similar, congruent)

(iii) All ________________ triangles are similar. (isosceles, equilateral)

= **Two polygons of the same number of sides are similar, if (a) their corresponding angles are ________________ and (b) their corresponding sides are ________________. (equal, proportional)**

Ans. (i) similar

12 similar

13 equilateral

14 equal, proportional

18 Give two different examples of pair of:

(i) similar figures

(ii) non-similar figures

Ans. (i) Two different examples of a pair of similar figures are:

(iv) Any two rectangles

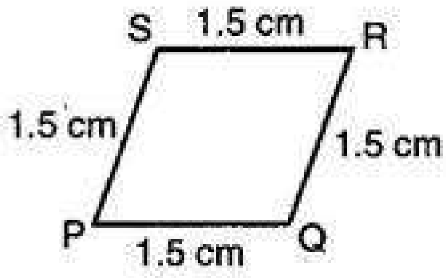
(v) Any two squares

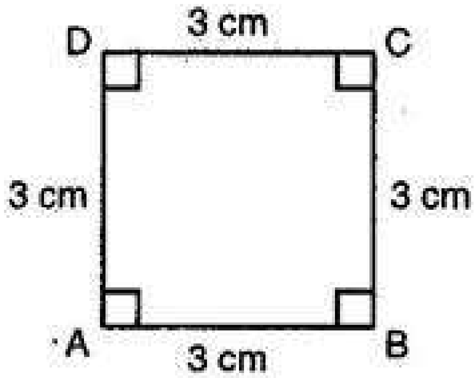
(ii) Two different examples of a pair of non-similar figures are:

(a) A scalene and an equilateral triangle

(b) An equilateral triangle and a right angled triangle

3. State whether the following quadrilaterals are similar or not:



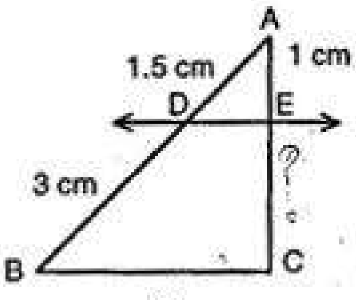


Ans. On looking at the given figures of the quadrilaterals, we can say that they are not similar because their angles are not equal.

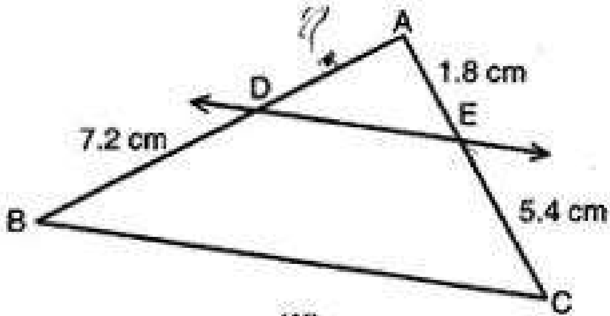
Chapter - 6

Triangles - Exercise 6.2

1. In figure (i) and (ii), DE $\parallel$ BC. Find EC in (i) and AD in (ii).



(i)



(ii)

Ans. (i) Since DE $\parallel$ BC,

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{1.5}{3} = \frac{1}{EC}$$

$$\Rightarrow EC = \frac{3}{1.5}$$

$$\Rightarrow EC = 2 \text{ cm}$$

(ii) Since DE $\parallel$ BC,

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4}$$

$\Rightarrow$ AD = $\dfrac{1.8\times 7.2}{5.4}$

$\Rightarrow$ EC = 2.4 cm

= **E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF $\parallel$ QR:**

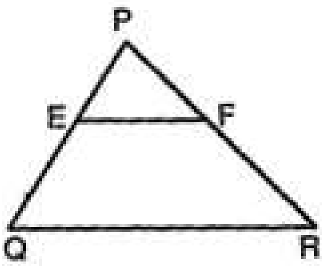
= **PE = 3.9 cm, EQ = 4 cm, PF = 3.6 cm and FR = 2.4 cm**

= **PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm**

= **PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm Ans.**

(i) Given: PE = 3.9 cm, EQ = 4 cm, PF = 3.6 cm and FR = 2.4 cm

Now, $\dfrac{PE}{EQ}=\dfrac{3.9}{4}$ = 0.97 cm



And $\dfrac{PF}{FR}=\dfrac{3.6}{2.4}$ = 1.2 cm

$\because \dfrac{PE}{EQ}\neq\dfrac{PF}{FR}$

Therefore, EF does not divide the sides PQ and PR of ΔPQR in the same ratio.

$\therefore$ EF is not parallel to QR.

(ii) Given: PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

Now, $\dfrac{PE}{EQ} = \dfrac{4}{4.5} = \dfrac{8}{9}$ cm

And $\dfrac{PF}{FR} = \dfrac{8}{9}$ cm

$\because \dfrac{PE}{EQ} = \dfrac{PF}{FR}$

Therefore, EF divides the sides PQ and PR of ΔPQR in the same ratio.

$\therefore$ EF is parallel to QR.

(iii) Given: PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm $\Rightarrow$

EQ = PQ – PE = 1.28 – 0.18 = 1.10 cm

And ER = PR – PF = 2.56 – 0.36 = 2.20 cm

Now, $\dfrac{PE}{EQ} = \dfrac{0.18}{1.10} = \dfrac{18}{110} = \dfrac{9}{55}$ cm

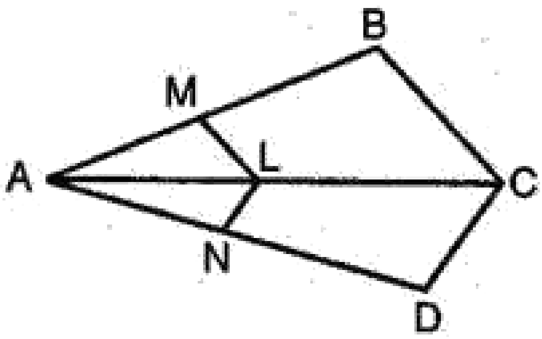
And $\dfrac{PF}{FR} = \dfrac{0.36}{2.20} = \dfrac{36}{220} = \dfrac{9}{55}$ cm

$\because \dfrac{PE}{EQ} = \dfrac{PF}{FR}$

Therefore, EF divides the sides PQ and PR of ΔPQR in the same ratio.

$\therefore$ EF is parallel to QR.

3. In figure, if LM $\parallel$ CB and LN $\parallel$ CD, prove that $\dfrac{AM}{AB} = \dfrac{AN}{AD}$.



Ans. In ΔABC, LM $\parallel$ CB

$\therefore \frac{AM}{AB} = \frac{AL}{AC}$ [Basic Proportionality theorem] ……….(i)

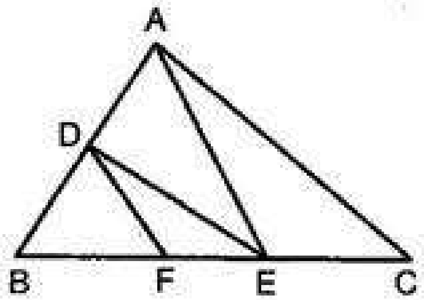
And in ΔACD, LN $\parallel$ CD

$\therefore \frac{AL}{AC} = \frac{AN}{AD}$ [Basic Proportionality theorem] ……….(ii)

From eq. (i) and (ii), we have

$$\frac{AM}{AB} = \frac{AN}{AD}$$

4. In the given figure, DE $\parallel$ AC and DF $\parallel$ AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.



Ans. In ΔBCA, DE $\parallel$ AC

$\therefore \frac{BE}{EC} = \frac{BD}{DA}$ [Basic Proportionality theorem] ……….(i)

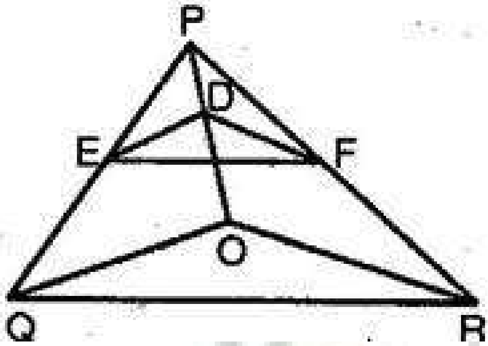
And in ΔBEA, DF $\parallel$ AE

$\therefore \frac{BE}{FE} = \frac{BD}{DA}$ [Basic Proportionality theorem] ……….(ii)

From eq. (i) and (ii), we have

$\frac{BF}{FE} = \frac{BE}{EC}$

5. In the given figure, DE $\parallel$ OQ and DF $\parallel$ OR. Show that EF $\parallel$ QR.



Ans. In ΔPQO, DE $\parallel$ OQ

$\therefore \frac{PE}{EQ} = \frac{PD}{DO}$ [Basic Proportionality theorem] ……….(i)

And in ΔPOR, DF $\parallel$ OR

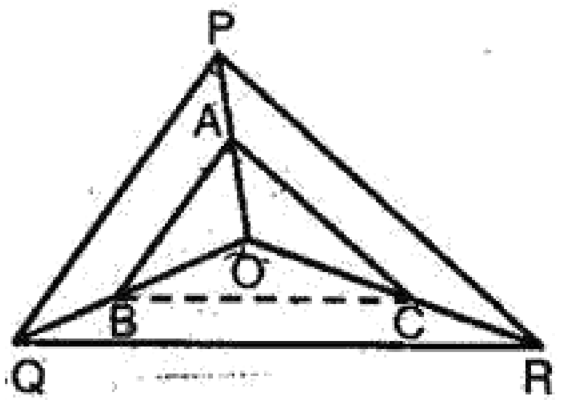
$\therefore \frac{PD}{DO} = \frac{PF}{FR}$ [Basic Proportionality theorem] ……….(ii)

From eq. (i) and (ii), we have

$\frac{PE}{EQ} = \frac{PF}{FR}$

$\therefore$ EF $\parallel$ QR [By the converse of BPT]

15 In the given figure, A, B, and C are points on OP, OQ and OR respectively such that AB $\parallel$ PQ and AC $\parallel$ PR. Show that BC $\parallel$ QR.



Ans. Given: O is any point in ΔPQR, in which AB $\parallel$ PQ and AC $\parallel$ PR.

To prove: BC $\parallel$ QR

Construction: Join BC.

Proof: In ΔOPQ, AB $\parallel$ PQ

$\therefore \frac{OA}{AP} = \frac{OB}{BQ}$ [Basic Proportionality theorem] ……….(i)

And in ΔOPR, AC $\parallel$ PR

$\therefore \frac{OA}{AP} = \frac{OC}{CR}$ [Basic Proportionality theorem] ……….(ii)

From eq. (i) and (ii), we have

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

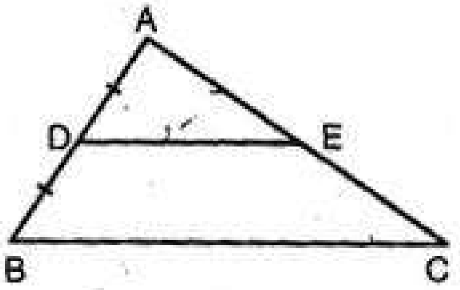
$\because$ In ΔOQR, B and C are points dividing the sides OQ and OR in the same ratio. $\therefore$ By the converse of Basic Proportionality theorem,

$\Rightarrow$ BC $\parallel$ QR

19 Using Theorem 6.1, prove that a line drawn through the midpoint of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Ans. Given: A triangle ABC, in which D is the midpoint of side AB and the

line DE is drawn parallel to BC, meeting AC at E.



To prove: AE = EC

Proof: Since DE $\parallel$ BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ [Basic Proportionality theorem] ……….(i)

But AD = DB [Given]

$\Rightarrow \frac{AD}{DB} = 1$

$\Rightarrow \frac{AE}{EC} = 1$ [From eq. (i)]

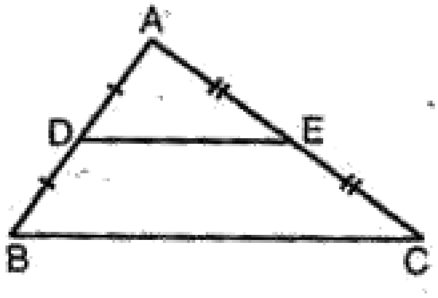
$\Rightarrow$ AE=EC

Hence, E is the midpoint of the third side AC.

(vi) Using Theorem 6.2, prove that the line joining the midpoints of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Ans. Given: A triangle ABC, in which D and E are the midpoints of

sides AB and AC respectively.



To Prove: DE $\parallel$ BC

Proof: Since D and E are the midpoints of AB and AC

respectively.

$\therefore$ AD = DB and AE = EC

Now, AD = DB

$\Rightarrow \frac{AD}{DB} = 1$ and AE = EC

$\Rightarrow \frac{AE}{EC} = 1$

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ = 1

$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$

Thus, in triangle ABC, D and E are points dividing the sides AB and AC in the same ratio. Therefore,

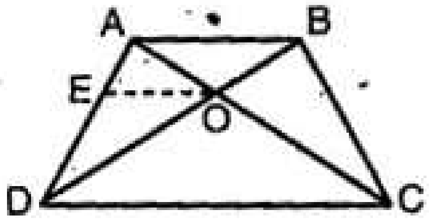
by the converse of Basic Proportionality theorem, we have

DE $\parallel$ BC

(iii) ABCD is a trapezium in which AB $\parallel$ DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Ans. Given: A trapezium ABCD, in which AB $\parallel$ DC and its diagonals AC and

BD intersect each other at O.



To Prove: $\frac{AO}{BO} = \frac{CO}{DO}$

Construction: Through O, draw OE $\parallel$ AB, i.e. OE $\parallel$ DC.

Proof: In ΔADC, we have OE $\parallel$ DC

$\therefore \frac{AE}{ED} = \frac{AO}{CO}$ [By Basic Proportionality theorem]……….(i)

Again, in ΔABD, we have OE $\parallel$ AB[Construction]

$\therefore \frac{ED}{AE} = \frac{DO}{BO}$ [By Basic Proportionality theorem]

$\Rightarrow \frac{AE}{ED} = \frac{BO}{DO}$ ……….(ii)

From eq. (i) and (ii), we get

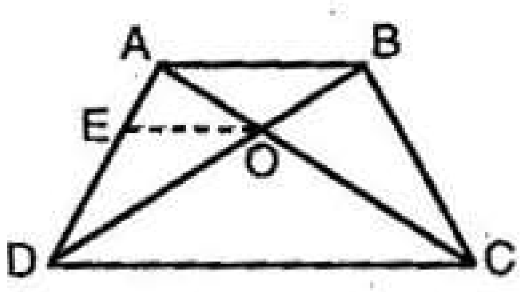
$\frac{AO}{CO} = \frac{BO}{DO}$

$\Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$

(c) The diagonals of a quadrilateral ABCD intersect each other at the point O such that Show that $\frac{AO}{BO} = \frac{CO}{DO}$. **ABCD is a trapezium.**

Ans. Given: A quadrilateral ABCD, in which its diagonals AC and

BD intersect each other at O such that $\frac{AO}{BO}=\frac{CO}{DO}$, i.e.



$\frac{AO}{CO}=\frac{BO}{DO}$.

To Prove: Quadrilateral ABCD is a trapezium.

Construction: Through O, draw OE $\parallel$ AB meeting AD at E.

Proof: In ΔADB, we have OE $\parallel$ AB [By construction]

$\therefore \frac{DE}{EA}=\frac{OD}{BO}$ [By Basic Proportionality theorem]

$\Rightarrow \frac{EA}{DE}=\frac{BO}{DO}$

$\Rightarrow \frac{EA}{DE}=\frac{BO}{DO}=\frac{AO}{CO}$

$\left[\because \frac{AO}{CO}=\frac{BO}{DO}\right]$

$\Rightarrow \frac{EA}{DE}=\frac{AO}{CO}$

Thus in ΔADC, E and O are points dividing the sides AD and AC in the same ratio. Therefore by the converse of Basic Proportionality theorem, we have

EO $\parallel$ DC

But EO $\parallel$ AB[By construction]

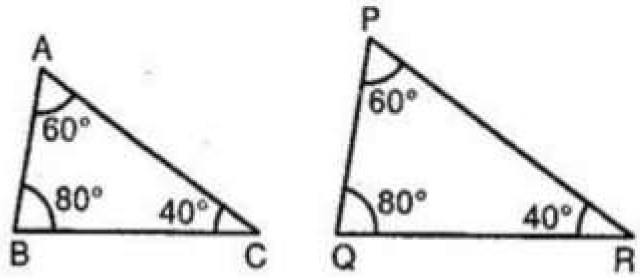
$\therefore$ AB $\parallel$ D

$\therefore$ Quadrilateral ABCD is a trapezium

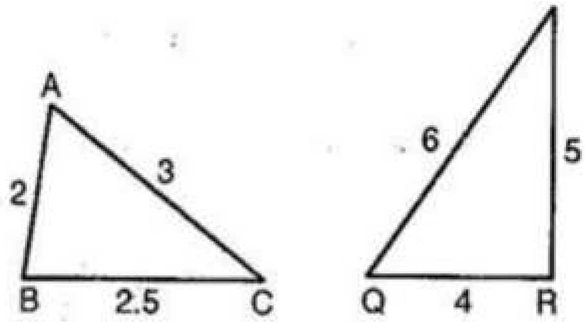
Chapter - 6

Triangles - Exercise 6.3

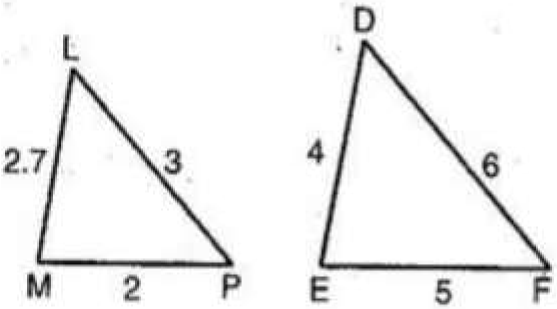
= **State which pairs of triangles in the given figure, are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:**



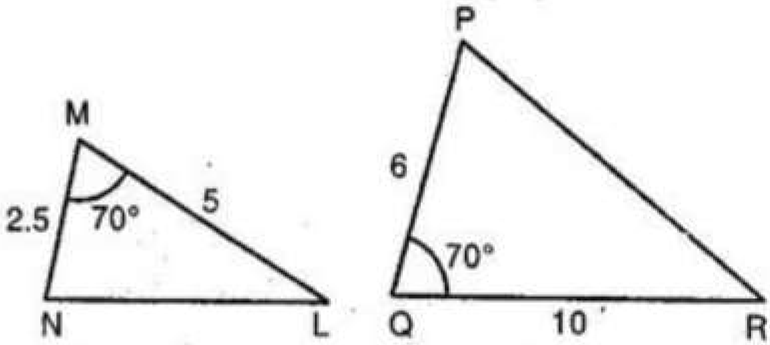
(i)



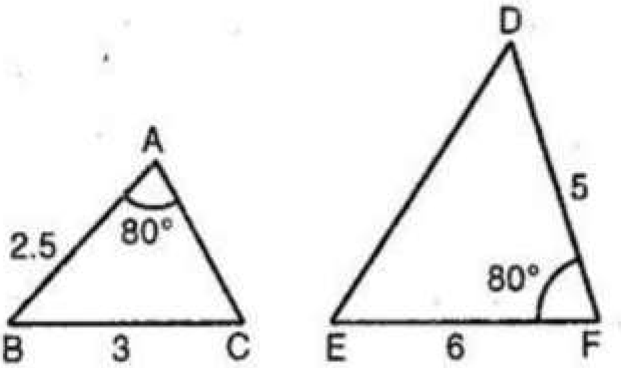
(ii)



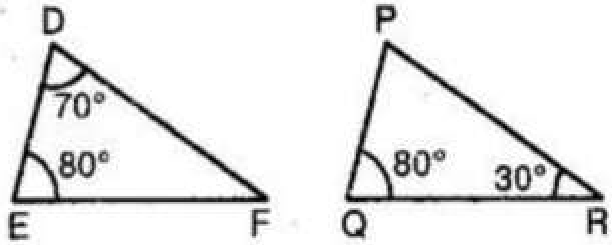
(iii)



(iv)



(v)



(vi)

Ans. (i) In Δs ABC and PQR, we observe that,

$\angle A = \angle P = 60^\circ$, $\angle B = \angle Q = 80^\circ$ and $\angle C = \angle R = 40^\circ$

$\therefore$ By AAA criterion of similarity, $\Delta ABC \sim \Delta PQR$

= In Δs ABC and PQR, we observe that,

$$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ} = \frac{1}{2}$$

$\therefore$ By SSS criterion of similarity, $\Delta ABC \sim \Delta PQR$

16 In Δs LMP and DEF, we observe that the ratio of the sides of these triangles is not equal. Therefore, these two triangles are not similar.

17 In Δs MNL and QPR, we observe that, $\angle M = \angle Q = 70^\circ$

But, $\frac{MN}{PQ} \neq \frac{ML}{QR}$

$\therefore$ These two triangles are not similar as they do not satisfy SAS criterion of similarity.

20 In Δs ABC and FDE, we have, $\angle A = \angle F = 80^\circ$

But, $\frac{AB}{DE} \neq \frac{AC}{DF}$ [$\because$ AC is not given]

$\therefore$ These two triangles are not similar as they do not satisfy SAS criterion of similarity.

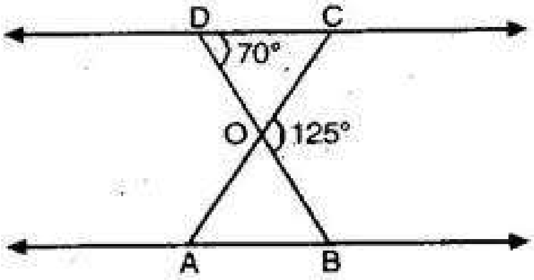
(vii) In Δs DEF and PQR, we have, $\angle D = \angle P = 70^\circ$

[$\because \angle P = 180^\circ - 80^\circ - 30^\circ = 70^\circ$]

And $\angle E = \angle Q = 80^\circ$

$\therefore$ By AAA criterion of similarity, $\Delta DEF \sim \Delta PQR$

(iv) In figure, ΔODC $\sim \Delta$ OBA, $\angle$ BOC = $125°$ and $\angle$ CDO = $70°$. Find $\angle$ DOC, $\angle$ DCO and $\angle$ OAB.



Ans. Since BD is a line and OC is a ray on it.

$\therefore \angle DOC + \angle BOC = 180°$

$\Rightarrow \angle DOC + 125° = 180°$

$\Rightarrow \angle DOC = 55°$

In Δ CDO, we have $\angle CDO + \angle DOC + \angle DCO = 180°$

$\Rightarrow 70° + 55° + \angle DCO = 180°$

$\Rightarrow \angle DCO = 55°$

It is given that ΔODC $\sim \Delta$ OBA

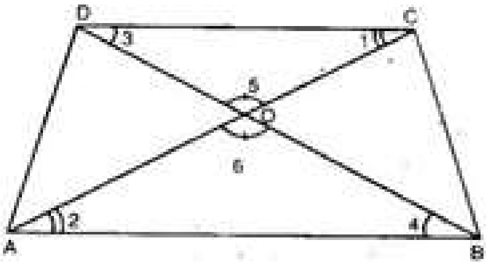
$\therefore \angle OBA = \angle ODC, \angle OAB = \angle OCD$

$\Rightarrow \angle OBA = 70°, \angle OAB = 55°$

Hence $\angle$ DOC = $55°$; $\angle$ DCO = $55°$ and $\angle$ OAB = $55°$

(d) Diagonals AC and BD of a trapezium ABCD with AB || CD intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Ans. Given: ABCD is a trapezium in which AB $\parallel$ DC.



To Prove: $\frac{OA}{OC} = \frac{OB}{OD}$

Proof: In Δs OAB and OCD, we have,

$\angle 5 = \angle 6$ [Vertically opposite angles]

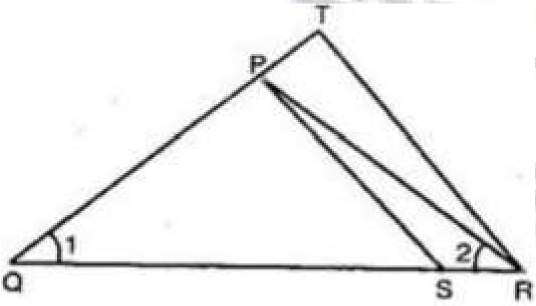
$\angle 1 = \angle 2$ [Alternate angles]

And $\angle 3 = \angle 4$ [Alternate angles]

$\therefore$ By AAA criterion of similarity, $\Delta OAB \sim \Delta ODC$

Hence, $\frac{OA}{OC} = \frac{OB}{OD}$

4. In figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.



Ans. We have, $\frac{QR}{QS} = \frac{QT}{PR}$

$\Rightarrow \frac{QT}{QR} = \frac{PR}{QS}$ ……….(1)

Also, $\angle 1 = \angle 2$ [Given]

$\therefore$ PR = PQ ……….(2) [$\because$ Sides opposite to equal $\angle$s are equal]

From eq.(1) and (2), we get

$$\frac{QT}{QR} = \frac{PR}{QS} \Rightarrow \frac{PQ}{QT} = \frac{QS}{QR}$$

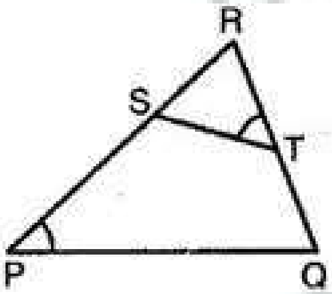
In Δs PQS and TQR, we have,

$\frac{PQ}{QT} = \frac{QS}{QR}$ and $\angle$PQS = $\angle$TQR = $\angle$Q

$\therefore$ By SAS criterion of similarity, ΔPQS ~ ΔTQR

5. S and T are points on sides PR and QR of a ΔPQR such that $\angle$P = $\angle$RTS. Show that ΔRPQ ~ ΔRTS.

Ans. In Δs RPQ and RTS, we have



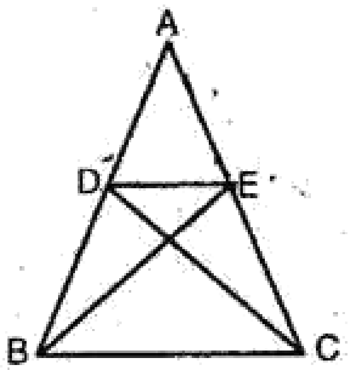
$\angle$RPQ = $\angle$RTS [Given]

$\angle$PRQ = $\angle$TRS [Common]

$\therefore$ By AA-criterion of similarity,

ΔRPQ ~ ΔRTS

6. In the given figure, if ΔABE $\cong$ ΔACD, show that ΔADE ~ ΔABC.



Ans. It is given that $\Delta ABE \cong \Delta ACD$

$\therefore$ AB = AC and AE = AD

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$$

$$\Rightarrow \frac{AB}{AC} = \frac{AD}{AE} \text{ ....(1)}$$

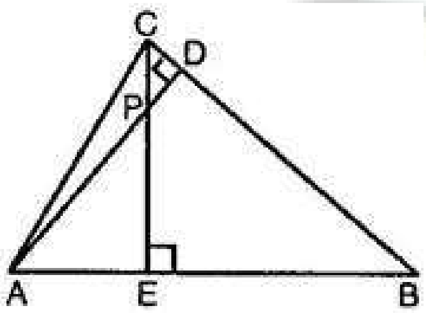
$\therefore$ In Δs ADE and ABC, we have,

$\frac{AB}{AC} = \frac{AD}{AE}$ [from eq.(1)]

And $\angle$ BAC = $\angle$ DAE [Common]

Thus, by SAS criterion of similarity, $\Delta ADE \sim \Delta ABC$

7. In figure, altitude AD and CE of a ΔABC intersect each other at the point P. Show that:



(i) ΔAEP $\sim$ ΔCDP

(ii) Δ**ABD** $\sim$ Δ **CBE**

(iii) Δ**AEP** $\sim$ Δ **ADB**

(iv) Δ**PDC** $\sim$ Δ **BEC**

Ans. (i) In Δs AEP and CDP, we have,

$\angle$ AEP = $\angle$ CDP = 90° [$\because$ CE $\perp$ AB, AD $\perp$ BC]

And $\angle$ APE = $\angle$ CPD[Vertically opposite]

$\therefore$ By AA-criterion of similarity, ΔAEP $\sim$ Δ CDP

(ii) In Δs ABD and CBE, we have,

$\angle$ ADB = $\angle$ CEB = 90°

And $\angle$ ABD = $\angle$ CBE[Common]

$\therefore$ By AA-criterion of similarity, ΔABD $\sim$ Δ CBE

(iii) In Δs AEP and ADB, we have,

$\angle$ AEP = $\angle$ ADB = 90° [$\because$ AD $\perp$ BC, CE $\perp$ AB]

And $\angle$ PAE = $\angle$ DAB[Common]

$\therefore$ By AA-criterion of similarity, ΔAEP $\sim$ Δ ADB

(iv) In Δs PDC and BEC, we have,

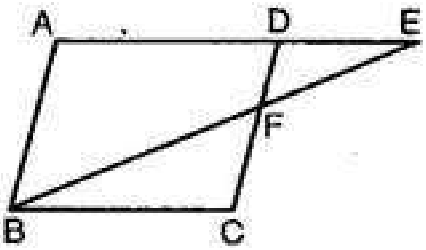
$\angle$ PDC = $\angle$ BEC = 90° [$\because$ CE $\perp$ AB, AD $\perp$ BC]

And $\angle$ PCD = $\angle$ BEC[Common]

$\therefore$ By AA-criterion of similarity, ΔPDC $\sim$ Δ BEC

8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that Δ**ABE** $\sim$ Δ **CFB.**

Ans. In Δs ABE and CFB, we have,



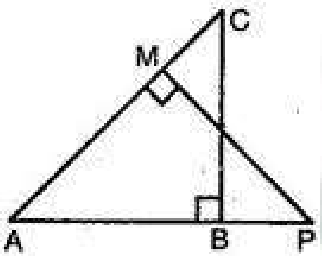
$\angle$ AEB = $\angle$ CBF[Alt. $\angle$ s]

$\angle$ A = $\angle$ C [opp. $\angle$ s of a $\parallel$ gm]

$\therefore$ By AA-criterion of similarity, we have

$\Delta_{ABE} \sim \Delta_{CFB}$

9. In the given figure, ABC and AMP are two right triangles, right angles at B and M respectively. Prove that:



(i) Δ **ABC** $\sim \Delta$ **AMP**

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Ans. (i) In Δs ABC and AMP, we have,

$\angle$ ABC = $\angle$ AMP = 90° [Given]

$\angle$ BAC = $\angle$ MAP [Common angles]

$\therefore$ By AA-criterion of similarity, we have

$\Delta_{ABC} \sim \Delta_{AMP}$

(ii) We have ΔABC ~ ΔAMP [As prove above]

$$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP}$$

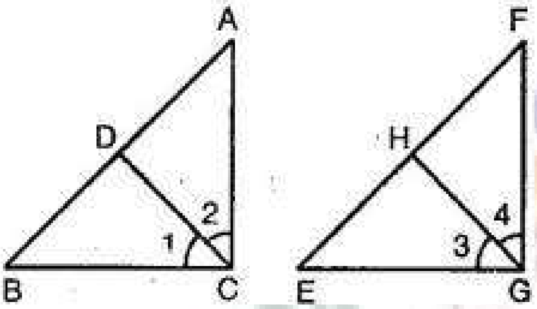
10. CD and GH are respectively the bisectors of $\angle$ACB and $\angle$EGF such that D and H lie on sides AB and FE at ΔABC and ΔEFG respectively. If ΔABC ~ ΔFEG, show that:

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) **ΔDCB ~ ΔHE**

(iii) **ΔDCA ~ ΔHGF**

Ans. We have, ΔABC ~ ΔFEG



$\Rightarrow \angle$A = $\angle$F………(1)

And $\angle$C = $\angle$G

$\Rightarrow \frac{1}{2}\angle C = \frac{1}{2}\angle G$

$\Rightarrow \angle$1 = $\angle$3 and $\angle$2 = $\angle$4 ……….(2)

[$\because$ CD and GH are bisectors of $\angle$C and $\angle$G

respectively]

$\therefore$ In Δs DCA and HGF, we have

$\angle$A = $\angle$F[From eq.(1)]

$\angle 2 = \angle 4$[From eq.(2)]

$\therefore$ By AA-criterion of similarity, we have

$\Delta DCA \sim \Delta HGF$

Which proves the (iii) part

We have, $\Delta DCA \sim \Delta HGF$

$$\Rightarrow \frac{AG}{FG} = \frac{CD}{GH}$$

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

Which proves the (i) part

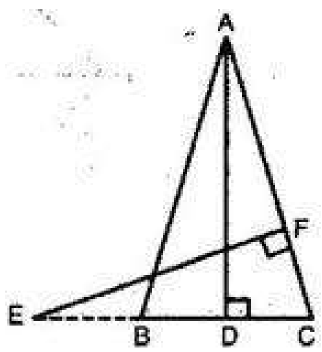
In Δs DCA and HGF, we have

$\angle 1 = \angle 3$[From eq.(2)]

$\angle B = \angle E$[$\because \Delta DCB \sim \Delta HE$]

Which proves the (ii) part

11. In the given figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD $\perp$ BC and EF $\perp$ AC, prove that $\Delta ABD \sim \Delta ECF$.



Ans. Here ΔABC is isosceles with AB = AC

$\therefore \angle B = \angle C$

In Δs ABD and ECF, we have

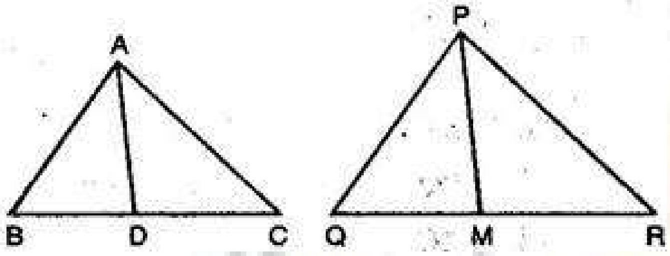
$\angle$ ABD = $\angle$ ECF[$\because$ $\angle$ B = $\angle$ C]

$\angle$ ABD = $\angle$ ECF = 90° [$\because$ AD $\perp$ BC and EF $\perp$ AC]

$\therefore$ By AA-criterion of similarity, we have

ΔABD ~ ΔECF

12. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of a ΔPQR (see figure). Show that ΔABC ~ ΔPQR.



Ans. Given: AD is the median of Δ ABC and PM is the median of Δ PQR such that

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

To prove: Δ ABC ~ Δ PQR

Proof: BD = $\frac{1}{2}$BC [Given]

And QM = $\frac{1}{2}$QR [Given]

Also $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$ [Given]

$$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AD}{PM}$$

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

$\therefore \Delta ABD \sim \Delta PQM$[By SSS-criterion of similarity]

$\Rightarrow \angle B = \angle Q$[Similar triangles have corresponding angles equal]

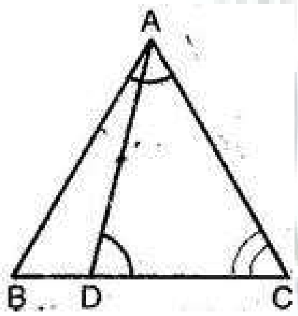
And $\frac{AB}{PQ} = \frac{BC}{QR}$[Given]

$\therefore$ By SAS-criterion of similarity, we have

$\Delta ABC \sim \Delta PQR$

13. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that CA^2 = CB.CD.

ANS. In triangles ABC and DAC,



$\angle ADC = \angle BAC$ [Given]

and $\angle C = \angle C$[Common]

$\therefore$ By AA-similarity criterion,

$\Delta ABC \sim \Delta DAC$

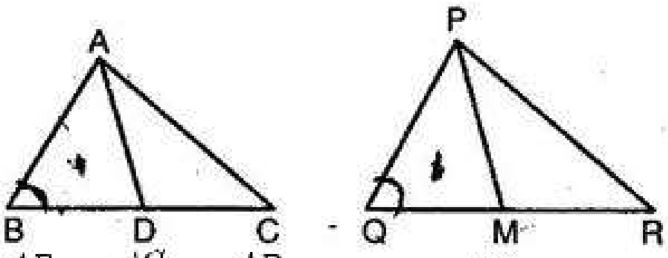
$\Rightarrow \frac{AB}{DA} = \frac{BC}{AC} = \frac{AC}{DC}$

$\Rightarrow \frac{CB}{CA} = \frac{CA}{CD}$

$\Rightarrow CA^2$ = CB.CD

14. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC $\sim \Delta$ PQR.

ANS. Given: AD is the median of Δ ABC and PM is the median of Δ PQR such that

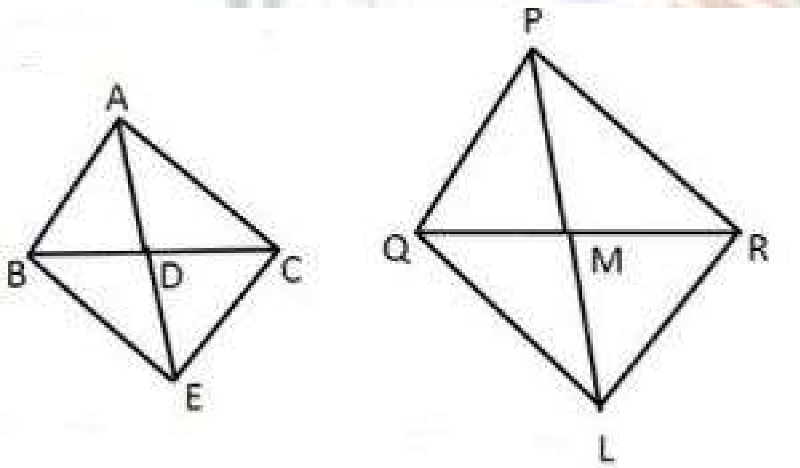


$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \quad \ldots\ldots\ldots(1)$$

To prove: Δ ABC $\sim \Delta$ PQR

Proof:

Let us extend AD to point D such that AD = DE and PM up to point L such that PM = ML



Join B to E. C to E,and Q to L, and R to L

We know that medians is the bisector of opposite side

Hence

BD=DC

Also, AD = DE (by construction)

Hence in quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

AC=BE

AB = EC (opposite sides of ||gm are equal) .... (2)

Similarly, we can prove that PQLR is a parallelogram

PR=QL

PQ = LR opposite sides of ||gm are equal) .... (3)

Given that

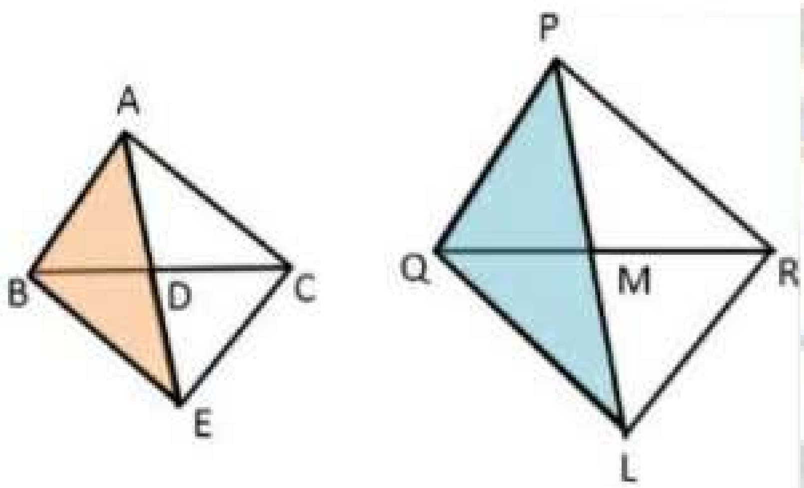
$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{AD}{PM}$ [from (2) (3)]

$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$

$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$ $[as\ AD = DE, AE = AD + DE = 2AD$
$PM = ML. PL = PM + ML = 2PM$

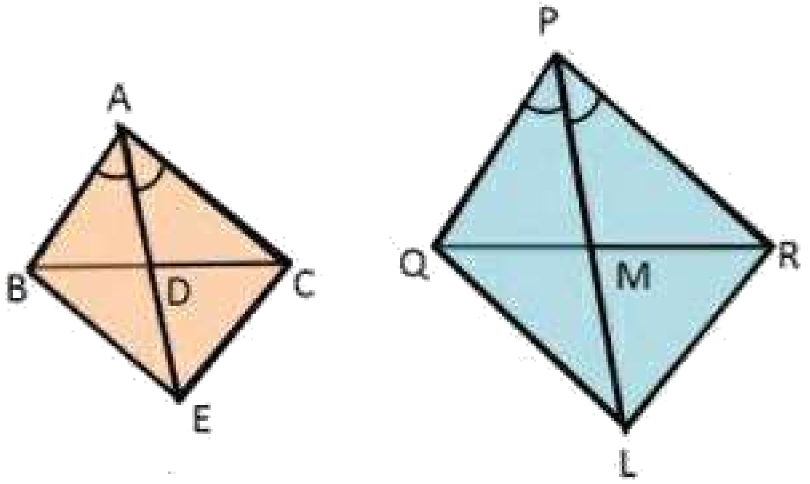
$\triangle ABE \sim \triangle PQL$ (By SSS Similiarity Criteria)



We know that corresponding angles of similar triangles are equal.

(4)

Similarly, we can prove that $\triangle AEC \sim \triangle PLR$.



We know that corresponding angles of similar triangles are equal.

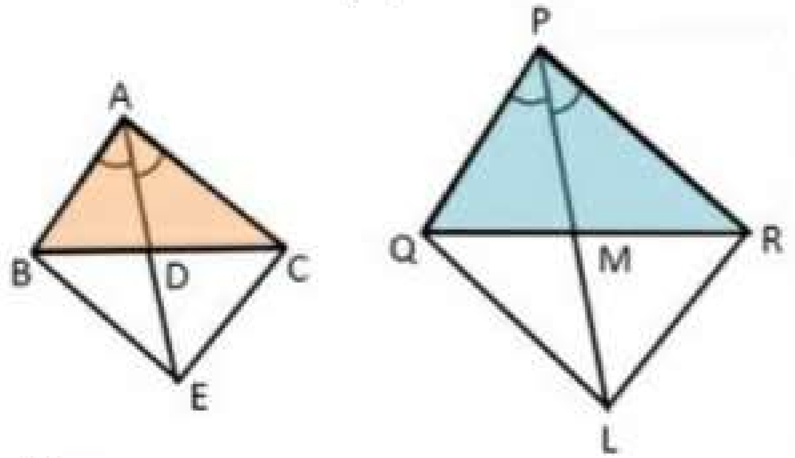
$\angle CAE = \angle RPL$ (5)

Adding (4) and (5),

$\angle BAE + \angle CAE = \angle QPL + \angle RPL$

$\angle CAB = \angle RPQ$

$In\ \triangle ABC\ and\ \triangle PQR,$



$\frac{AB}{PQ} = \frac{AC}{PR}$

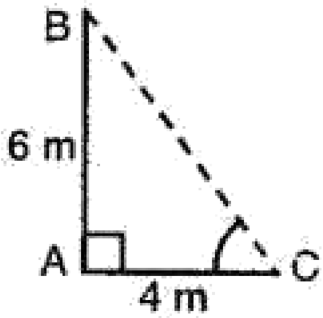
$\angle CAB = \angle RPQ$

$\triangle ABC \sim \triangle PQR$

Hence proved

15. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Ans. Let AB the vertical pole and AC be its shadow. Also, let DE be the vertical tower and DF be its shadow. Joined BC and EF.



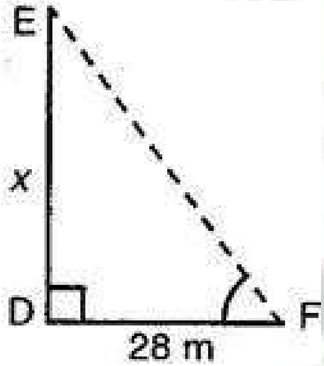
Let DE = x meters

Here, AB = 6 m, AC = 4 m and DF = 28 m

In the triangles ABC and DEF,

$\angle$A= $\angle$D= 90°

And $\angle$C = $\angle$F[Each is the angular elevation of the sun]



$\therefore$ By AA-similarity criterion,

ΔABC ~ ΔDEF

$$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF}$$

$$\Rightarrow \frac{6}{x} = \frac{4}{28}$$

$$\Rightarrow \frac{6}{x} = \frac{1}{7}$$

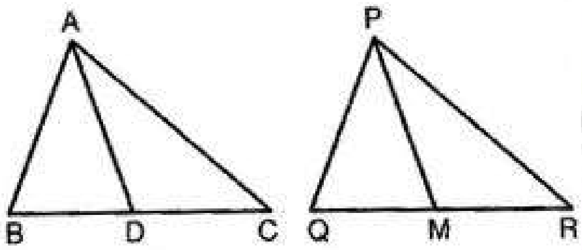
$\Rightarrow x$ = 42 m

Hence, the height of the tower is 42 meters.

16. If AD and PM are medians of triangles ABC and PQR respectively, where ΔABC $\sim\Delta$ PQR, prove that $\frac{AB}{PQ}=\frac{AD}{PM}$.

Ans. Given: AD and PM are the medians of triangles

ABC and PQR respectively, where



ΔABC $\sim\Delta$PQR

To prove: $\frac{AB}{PQ}=\frac{AD}{PM}$

Proof: In triangles ABD and PQM,

$\angle$B = $\angle$Q [Given]

And $\frac{AB}{PQ}=\frac{\frac{1}{2}BC}{\frac{1}{2}QR}$ [$\because$ AD and PM are the medians of BC and QR respectively]

$\Rightarrow \frac{AB}{PQ}=\frac{BD}{QM}$

$\therefore$ By SAS-criterion of similarity,

ΔABD $\sim\Delta$PQM

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$$

Chapter - 6

Triangles - Exercise 6.4

= **Let ΔABC ~ ΔDEF and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC.**

Ans. We have, $\dfrac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \dfrac{BC^2}{EF^2}$

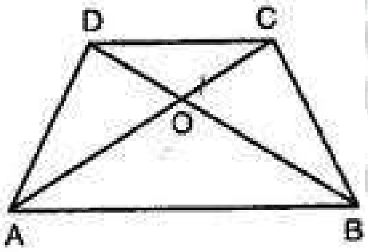
$$\Rightarrow \frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

$$\Rightarrow \frac{8}{11} = \frac{BC}{15.4}$$

$\Rightarrow$ BC = $\left(\dfrac{8}{11}\times 15.4\right)$ cm = 11.2 cm

= **Diagonals of a trapezium ABCD with AB $\parallel$ DC intersect each other at the point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD.**

Ans. In Δs AOB and COD, we have,



$\angle$AOB = $\angle$COD[Vertically opposite angles]

$\angle$OAB = $\angle$OCD[Alternate angles]

By AA-criterion of similarity,

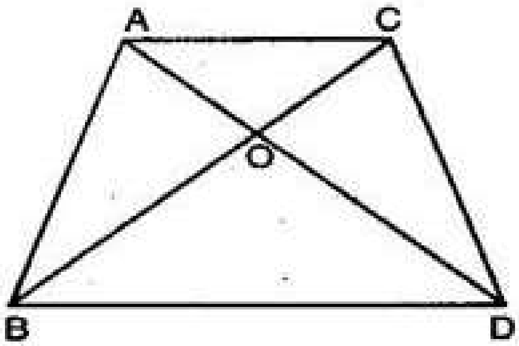
$\therefore \Delta_{AOB} \sim \Delta_{COD}$

$$\therefore \frac{\text{Area}(\Delta AOB)}{\text{Area}(\Delta COD)} = \frac{AB^2}{DC^2}$$

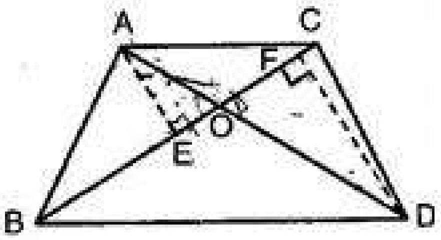
$$\Rightarrow \frac{\text{Area}(\Delta AOB)}{\text{Area}(\Delta COD)} = \frac{(2DC)^2}{DC^2} = \frac{4}{1}$$

Hence, Area (Δ AOB) : Area (Δ COD) = 4 : 1

18 In the given figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$.



Ans. Given: Two Δs ABC and DBC which stand on the same base but on the opposite sides of BC.



To Prove: $\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DBC)} = \frac{AO}{DO}$

Construction: Draw AE $\perp$ BC and DF $\perp$ BC.

Proof: In Δs AOE and DOF, we have, $\angle$ AEO = $\angle$ DFO = 90° and

$\angle$ AOE = $\angle$ DOF[Vertically opposite)

$\therefore$ $\Delta_{AOE} \sim \Delta_{DOF}$[By AA-criterion]

$\therefore$ $\dfrac{AE}{DF} = \dfrac{AO}{OD}$ ……….(i)

Now, $\dfrac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DBC)} = \dfrac{\frac{1}{2} \times BC \times AE}{\frac{1}{2} \times BC \times DF}$

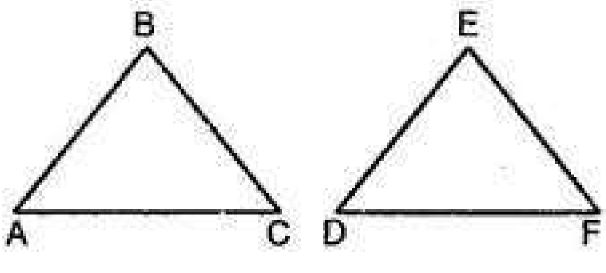
$\Rightarrow \dfrac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DBC)} = \dfrac{AE}{DF}$

$\Rightarrow \dfrac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DBC)} = \dfrac{AO}{OD}$ [using eq. (i)]

21If the areas of two similar triangles are equal, prove that they are congruent.

Ans. Given: Two Δs ABC and DEF such that $\Delta_{ABC} \sim \Delta_{DEF}$ And

Area(ΔABC) = Area (ΔDEF)



To Prove: $\Delta_{ABC} \cong \Delta_{DEF}$

Proof: $\Delta_{ABC} \sim \Delta_{DEF}$

$\therefore$ $\angle$ A= $\angle$ D, $\angle$ B= $\angle$ E, $\angle$ C= $\angle$ F

And $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

To establish ΔABC $\cong$ ΔDEF, it is sufficient to prove that, AB = DE, BC = EF and AC = DF Now,

Area(ΔABC) = Area (ΔDEF)

$\therefore \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = 1$

$\Rightarrow \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = 1$

$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = 1$

$\Rightarrow$ AB=DE,BC=EF,AC=DF

Hence, ΔABC $\cong$ ΔDEF

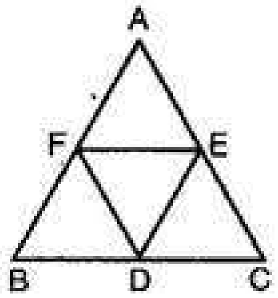
(viii) D, E and F are respectively the midpoints of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.

Ans. Since D and E are the midpoints of the sides BC and CA of ΔABC respectively.

$\therefore$ DE $\parallel$ BA $\Rightarrow$ DE $\parallel$ FA ……….(i)

Since D and F are the midpoints of the sides BC and AB of ΔABC respectively.

$\therefore$ DF $\parallel$ CA $\Rightarrow$ DE $\parallel$ AE ……….(ii)



From (i) and (ii), we can say that AFDE is a parallelogram.

Similarly, BDEF is a parallelogram.

Now, in Δs DEF and ABC, we have

$\angle$ FDE = $\angle$ A[opposite angles of $\parallel$ gm AFDE]

And $\angle$ DEF = $\angle$ B[opposite angles of $\parallel$ gm BDEF]

$\therefore$ By AA-criterion of similarity, we have ΔDEF ~ ΔABC

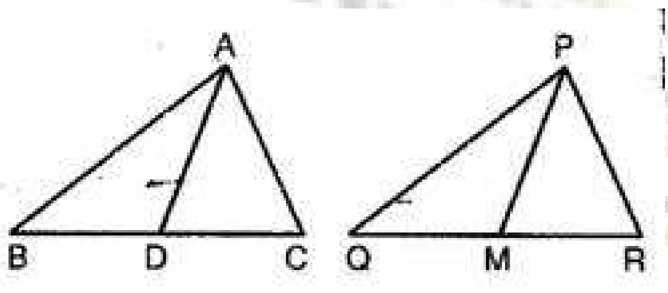
$$\Rightarrow \frac{\text{Area } (\Delta\text{DEF})}{\text{Area } (\Delta\text{ABC})} = \frac{\text{DE}^2}{\text{AB}^2} = \frac{\left(\frac{1}{2}\text{AB}\right)^2}{\text{AB}^2} \frac{1}{4}$$

[$\because$ DE= $\frac{1}{2}$AB]

Hence, Area (ΔDEF): Area (ΔABC) = 1 : 4

(v) Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Ans. Given: ΔABC ~ ΔPQR, AD and PM are the medians of Δs ABC and PQR respectively.



To Prove: $\frac{\text{Area } (\Delta\text{ABC})}{\text{Area } (\Delta\text{PQR})} = \frac{\text{AD}^2}{\text{PM}^2}$

Proof: Since ΔABC ~ ΔPQR

$\therefore \frac{\text{Area } (\Delta\text{ABC})}{\text{Area } (\Delta\text{PQR})} = \frac{\text{AB}^2}{\text{PQ}^2}$ ……….(1)

But, $\frac{AB}{PQ} = \frac{AD}{PM}$ ……….(2)

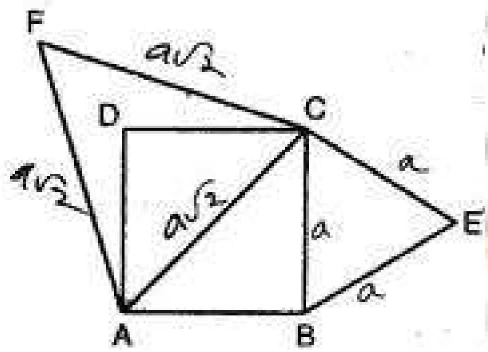
$\therefore$ From eq. (1) and (2), we have,

$$\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta PQR)} = \frac{AD^2}{PM^2}$$

(e) Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of the diagonals.

Ans. Given: A square ABCD,

Equilateral Δs BCE and ACF have been drawn on side BC and the diagonal AC respectively.



To Prove: Area (ΔBCE) = $\frac{1}{2}$ Area (ΔACF)

Proof: $\Delta BCE \sim \Delta ACF$

[Being equilateral so similar by AAA criterion of

similarity]

$$\Rightarrow \frac{\text{Area }(\Delta BCE)}{\text{Area }(\Delta ACF)} = \frac{BC^2}{AC^2}$$

$$\Rightarrow \frac{\text{Area }(\Delta BCE)}{\text{Area }(\Delta ACF)} = \frac{BC^2}{\left(\sqrt{2}BC\right)^2}$$

[$\because$ Diagonal = $\sqrt{2}$ side $\Rightarrow$ AC = $\sqrt{2}$ BC]

$$\Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta BDE)} = \frac{4}{1}$$

$$\Rightarrow \frac{}{\text{Area}(\Delta ACF)} = \frac{}{2}$$

Tick the correct answer and justify:

6. ABC and BDE are two equilateral triangles such that D is the midpoint of BC. The ratio of the areas of triangles ABC and BDE is:

8. 2: 1

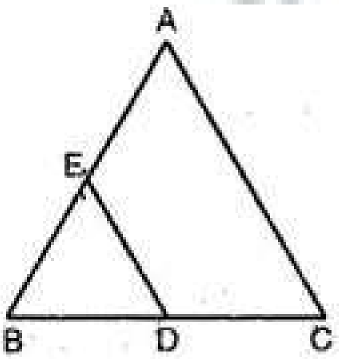
9. 1: 2

10. 4: 1

11. 1: 4

Ans. (C) Since ΔABC and ΔBDE are equilateral, they are equiangular and hence,

$\Delta_{ABC} \sim \Delta_{BDE}$



$$\Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta BDE)} = \frac{BC^2}{BD^2}$$

$$\Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta BDE)} = \frac{(2BD)^2}{BD^2}$$

[$\because$ D is the midpoint of BC]

$\therefore$ (C) is the correct answer.

(v)Sides of two similar triangles are in the ratio 4: 9. Areas of these triangles are in the ratio:

(iv) 2: 3

(v) 4: 9

(vi) 81: 16

(vii) 16: 81

Ans. (D) Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides. Therefore,

Ratio of areas = $\frac{(4)^2}{(9)^2} = \frac{16}{81}$

$\therefore$ (D) is the correct answer.

Chapter - 6

Triangles - Exercise 6.5

= **Sides of triangles are given below. Determine which of them right triangles are. In case of a right triangle, write the length of its hypotenuse.**

= **7 cm, 24 cm, 25 cm**

= **3 cm, 8 cm, 6 cm**

= **50 cm, 80 cm, 100 cm**

= **13 cm, 12 cm, 5 cm**

Ans. (i) Let a = 7 cm, b = 24 cm and c = 25 cm

Here the larger side is c = 25 cm.

We have, $a^2+b^2=7^2+24^2=49+576=625=c^2$

So, the triangle with the given sides is a right triangle. Its hypotenuse = 25 cm

19 Let a = 3 cm, b = 8 cm and c = 6 cm

Here the larger side is b = 8 cm.

We have, $a^2+c^2=3^2+6^2=9+36=45\neq b^2$

So, the triangle with the given sides is not a right triangle.

20 Let a = 50 cm, b = 80 cm and c = 100 cm

Here the larger side is c = 100 cm.

We have, $a^2+b^2=50^2+80^2=2500+6400=8900\neq c^2$

So, the triangle with the given sides is not a right triangle.

22 Let a = 13 cm, b = 12 cm and c = 5 cm

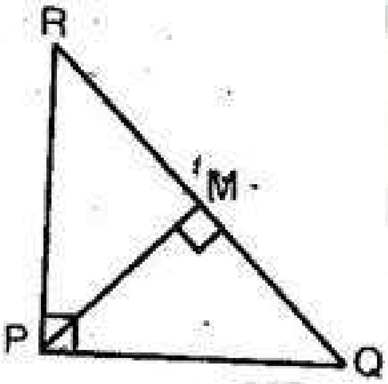
Here the larger side is a = 13 cm.

We have, $b^2 + c^2 = 12^2 + 5^2 = 144 + 25 = 169 = a^2$

So, the triangle with the given sides is a right triangle. Its hypotenuse = 13 cm

(ix) PQR is a triangle right angled at P and M is a point on QR such that PM $\perp$ QR. Show that PM^2 = QM x MR.

Ans. Given: PQR is a triangle right angles at P and PM $\perp$ QR



To Prove: PM^2 = QM.MR

Proof: Since PM $\perp$ QR

$\therefore \Delta QMP \sim \Delta PMR$

$\Rightarrow \frac{QM}{PM} = \frac{PM}{RM}$

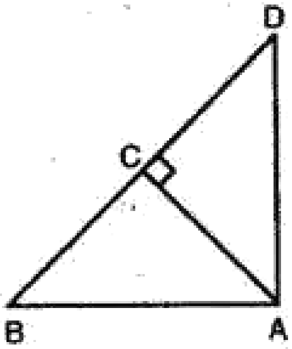
$\Rightarrow PM^2 = QM.MR$

(vi) In the given figure, ABD is a triangle right angled at A and AC $\perp$ BD. Show that:

(i) AB^2 = BC.BD

(ii) AC^2 = BC.DC

(iii) AD^2 = BD.CD



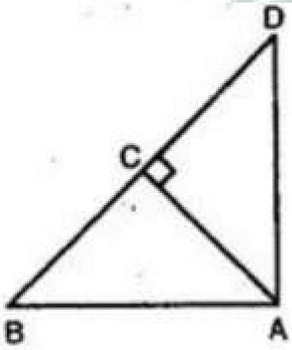
Ans. Given: ABD is a triangle right angled at A and AC $\perp$ BD.

To Prove: **(i)** $AB^2 = BC.BD$, **(ii)** $AC^2 = BC.DC$, **(iii)** $AD^2 = BD.CD$

Proof:**(i)** Since AC $\perp$ BD

$\therefore$ ΔCBA $\sim$ Δ CAD and each triangle is similar to ΔABD

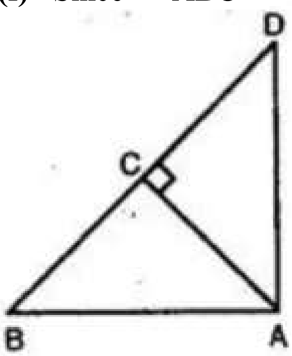
$\therefore$ ΔABD $\sim$ Δ CBA



$$\Rightarrow \frac{AB}{BD} = \frac{BC}{AB}$$

$$\Rightarrow AB^2 = BC.BD$$

(f) Since ΔABC $\sim$ Δ DAC



$$\Rightarrow \frac{AC}{BC} = \frac{DC}{AC}$$

$\Rightarrow AC^2 = BC.DC$

7. Since ΔCAD ~ ΔABD

$$\Rightarrow \frac{AD}{CD} = \frac{BD}{AD}$$

$\Rightarrow AD^2 = BD.CD$

12. ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Ans. Since ABC is an isosceles right triangle, right angled at C.

$\Rightarrow AB^2 = AC^2 + BC^2$
$\Rightarrow AB^2 = AC^2 + AC^2$ [BC = AC, given]
$\Rightarrow AB^2 = 2AC^2$

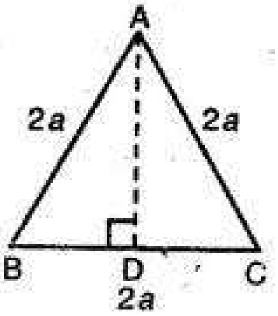
(vi) ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Ans. Since ABC is an isosceles right triangle with AC = BC and $AB^2 = 2AC^2$
$\Rightarrow AB^2 = AC^2 + AC^2$
$\Rightarrow AB^2 = AC^2 + BC^2$ [BC = AC, given]

$\therefore \Delta$ABC is right angled at C.

(viii) ABC is an equilateral triangle of side $2a$. Find each of its altitudes.

Ans. Let ABC be an equilateral triangle of side $2a$ units.



Draw AD $\perp$ BC. Then, D is the midpoint of BC.

$\Rightarrow$ BD= $\frac{1}{2}$ BC= $\frac{1}{2}\times 2a = a$

Since, ABD is a right triangle, right triangle at D.

$\therefore AB^2 = AD^2 + BD^2$

$\Rightarrow (2a)^2 = AD^2 + (a)^2$

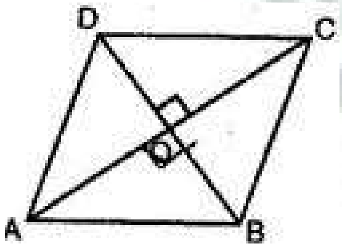
$\Rightarrow AD^2 = 4a^2 - a^2 = 3a^2$

$\therefore$ Each of its altitude = $\sqrt{3}a$

(v) Prove that the sum of the squares of the sides of a rhombus is equal to the sum of squares of its diagonals.

Ans. Let the diagonals AC and BD of rhombus ABCD intersect each other at O. Since the diagonals of a rhombus bisect each other at right angles.

$\therefore \angle AOB = \angle BOC = \angle COD = \angle DOA = 90^\circ$ and AO = CO, BO = OD



Since AOB is a right triangle, right angled at O.

$\therefore AB^2 = OA^2 + OB^2$

$\Rightarrow AB^2 = \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2$

[$\because$ OA = OC and OB = OD]

$\Rightarrow 4AB^2 = AC^2 + BD^2$ ……….(1)

……….(2)

Similarly, we have

$4CD^2 = AC^2 + BD^2$ ……….(3)

$4AD^2 = AC^2 + BD^2$ ……….(4)

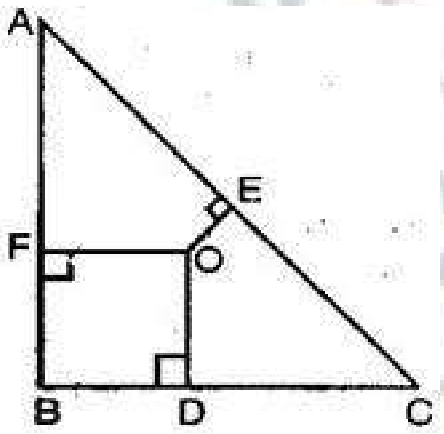
Adding all these results, we get

$4\left(AB^2 + BC^2 + CD^2 + DA^2\right) =$

$4\left(AC^2 + BD^2\right) \Rightarrow AB^2 + BC^2 + CD^2 + DA^2 =$

$AC^2 + BD^2$

9. In the given figure, O is a point in the interior of a triangle ABC, OD $\perp$ BC, OE $\perp$ AC and OF $\perp$ AB. Show that:



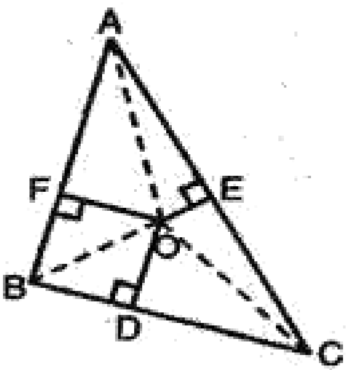
(i) $\mathbf{OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2}$

(ii) $\mathbf{AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2}$

Ans. Join AO, BO and CO.

10. In right Δs OFA, ODB and OEC, we have

$OA^2 = AF^2 + OF^2$, $OB^2 = BD^2 + OD^2$ and $OC^2 = CE^2 + OE^2$



Adding all these, we get

$$OA^2 + OB^2 + OC^2 = AF^2 + BD^2 + CE^2 + OF^2 + OD^2 + OE^2$$

$$\Rightarrow OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$$

(iii) In right Δs ODB and ODC, we have

$OB^2 = BD^2 + OD^2$ and

$OC^2 = OD^2 + CD^2 \Rightarrow$
$OB^2 - OC^2 = BD^2 - CD^2$ ……….(1)

Similarly, we have $OB^2 - OC^2 = BD^2 - CD^2$ ……….(2)

and $OB^2 - OC^2 = BD^2 - CD^2$ ……….(3)

Adding equations (1), (2) and (3), we get

$$= \left(OB^2 - OC^2\right) + \left(OC^2 - OA^2\right) + \left(OA^2 - OB^2\right)$$

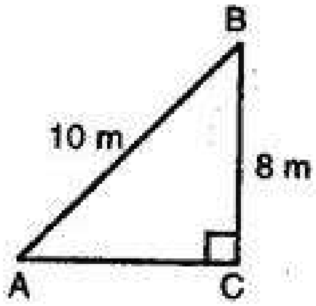
$$= \left(BD^2 - CD^2\right) + \left(CE^2 - AE^2\right) + \left(AF^2 - BF^2\right)$$

$$\Rightarrow \left(BD^2 + CE^2 + AF^2\right) - \left(AE^2 + CD^2 + BF^2\right) = 0$$

$$\Rightarrow AF^2 + BD^2 + CE^2 = AE^2 + BF^2 + CD^2$$

(iii) A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from the base of the wall.

Ans. Let AB be the ladder, B be the window and CB be the wall. Then, ABC is a right triangle, right angled at C.



$\Rightarrow AB^2 = AC^2 + BC^2$

$\Rightarrow 10^2 = AC^2 + 8^2$

$\Rightarrow AC^2 = 100 - 64$

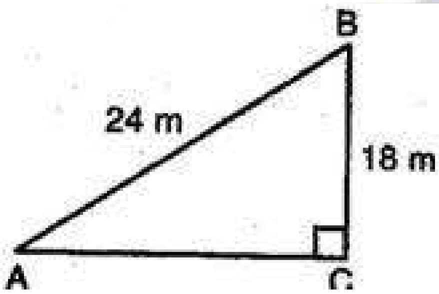
$\Rightarrow AC^2 = 36$

$\Rightarrow AC = 6$

Hence, the foot of the ladder is at a distance 6 m from the base of the wall.

11. A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other hand. How far from the base of the pole should the stake be driven so that the wire will be taut?

Ans. Let AB (= 24m) be a guy wire attached to a vertical pole. BC of height 18 m. To keep the wire taut, let it be fixed to a stake at A. Then, ABC is a right triangle, right angled at C.



$\therefore AB^2 = AC^2 + BC^2$

$\Rightarrow 24^2 = AC^2 + 18^2$

$\Rightarrow AC^2 = 576 - 324$

$\Rightarrow AC^2 = 252$

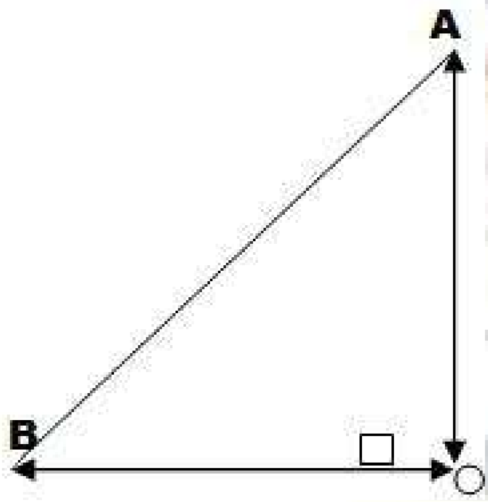
$\Rightarrow AC = 6\sqrt{7}$

Hence, the stake may be placed at a distance of $6\sqrt{7}$ m from the base of the pole.

(iv) An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Ans. Let the first aeroplane starts from O and goes upto A towards north where

OA = $\left(1000 \times \frac{3}{2}\right)$ km = 1500 km



Let the second aeroplane starts from O at the same time and goes upto B towards west where

OB = $\left(1200 \times \frac{3}{2}\right)$ km = 1800 km

According to the question the required distance = BA

In right angled triangle ABC, by Pythagoras theorem, we have,

$AB^2 = OA^2 + OB^2$

$= (1500)^2 + (1800)^2$

12. 2250000 + 3240000

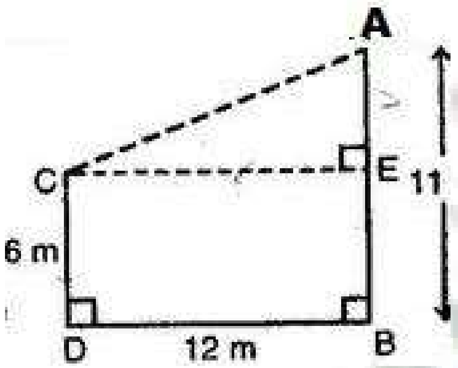
13. 5490000 =

$9 \times 61 \times 100 \times 100 \Rightarrow$ AB =

$300\sqrt{61}$ km

13. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Ans. Let AB = 11 m and CD = 6 m be the two poles such that BD = 12 m



Draw CE $\perp$ AB and join AC.

$\therefore$ CE = DB = 12 m

AE = AB – BE = AB – CD = (11 – 6)m = 5 m

In right angled triangle ACE, by Pythagoras theorem, we have

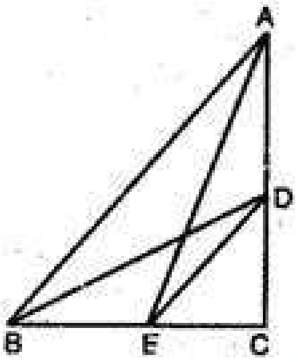
$AC^2 = CE^2 + AE^2 = 12^2 + 5^2$
= 144 + 25 = 169
AC = 13m

Hence, the distance between the tops of the two poles is 13 m.

15. D and E are points on the sides CA and CB respectively of a triangle ABC right angled

at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Ans. In right angled Δs ACE and DCB, we have



$AE^2 = AC^2 + CE^2$ and

$BD^2 = DC^2 + BC^2 \Rightarrow AE^2 + BD^2 = C^2 + BC^2)$

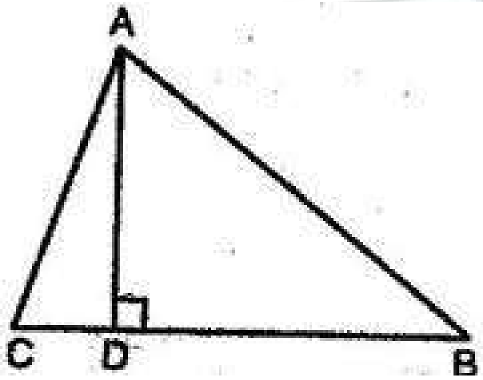
$\Rightarrow AE^2 + BD^2 =$

$\left(AC^2 + BC^2\right) + \left(DC^2 + CE^2\right) \Rightarrow AE^2 + BD^2 =$

$AB^2 + DE^2$

[By Pythagoras theorem, $AC^2 + BC^2 = AB^2$ and $DC^2 + CE^2 = DE^2$]

16. The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3CD (see figure). Prove that $2AB^2 = 2AC^2 + BC^2$.



Ans. We have, DB = 3CD

Now, BC = DB + CD

$\Rightarrow$ BC=3CD+CD

$\Rightarrow$ BC = 4CD

$\therefore$ CD = $\frac{1}{4}$BC and DB = 3CD = $\frac{3}{4}$BC ……….(1)

Since, Δ ABD is a right triangle, right angled at D. Therefore by Pythagoras theorem, we have,

$AB^2 = AD^2 + DB^2$ ……….(2)

Similarly, from ΔACD, we have, $AC^2 = AD^2 + CD^2$ ……….(3)

From eq. (2) and (3) $AB^2 - AC^2 = DB^2 - CD^2$

$\Rightarrow AB^2 - AC^2 = \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2$ [Using eq.(1)]

$\Rightarrow AB^2 - AC^2 = \left(\frac{9}{16} - \frac{1}{16}\right)BC^2$

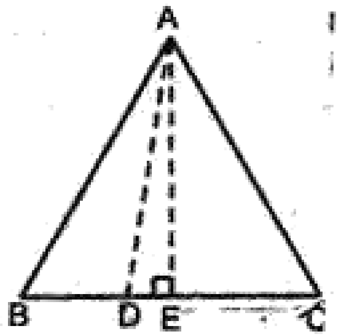
$\Rightarrow$ $AB^2 - AC^2 = \frac{1}{2}BC^2$

$\Rightarrow 2AB^2 - 2AC^2 = BC^2$

$\Rightarrow 2AB^2 = 2AC^2 + BC^2$

15. In an equilateral triangle ABC, D is a point on side BC such that BD = $\frac{1}{3}$BC. Prove that $9AD^2 = 7AB^2$.

Ans. Let ABC be an equilateral triangle and let D be a point on BC such that BD = $\frac{1}{3}$BC



Draw AE $\perp$ BC, Join AD.

In Δs AEB and AEC, we have,

AB = AC[$\because$ ΔABC is equilateral]

$\angle$ AEB = $\angle$ AEC [$\because$ each 90°]

And AE = AE

$\therefore$ By SAS-criterion of similarity, we have

ΔAEB ~ ΔAEC

$\Rightarrow$ BE=EC

Thus, we have, BD = $\frac{1}{3}$BC, DC = $\frac{2}{3}$BC and BE = EC = $\frac{1}{3}$BC ………(1)

Since, $\angle$ C = 60°

$\therefore$ ΔADC is an acute angle triangle.

$\therefore$ $AD^2 = AC^2 + DC^2 - 2DC \times EC$

= $AC^2 + \left(\frac{2}{3}\text{BC}\right)^2 - 2 \times \frac{2}{3}\text{BC} \times \frac{1}{2}\text{BC}$[using eq.(1)]

$\Rightarrow AD^2 = AC^2 + \frac{4}{9}\text{BC}^2 - \frac{2}{3}\text{BC}^2$

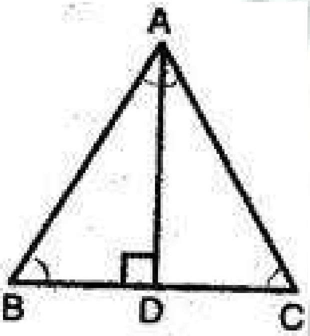
$= AB^2 + \frac{4}{9}AB^2 - \frac{2}{3}AB^2$ [$\because$ AB=BC=AC]

$\Rightarrow AD^2 = \frac{(9+4-6)AB^2}{9} = \frac{7}{9}AB^2$

$\Rightarrow 9AD^2 = 7AB^2$

17. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Ans. Let ABC be an equilateral triangle and let AD $\perp$ BC. In Δs ADB and ADC, we have,



AB = AC [Given]

$\angle$B = $\angle$C = 60° [Given]

And $\angle$ADB = $\angle$ADC[Each = 90°]

$\therefore \Delta$ADB $\cong \Delta$ADC[By RHS criterion of congruence]

$\Rightarrow$ BD=DC

$\Rightarrow$ BD=DC= $\frac{1}{2}$BC

Since ΔADB is a right triangle, right angled at D, by Pythagoras theorem, we have,

$AB^2 = AD^2 + BD^2$

$\Rightarrow AB^2 = AD^2 + \left(\frac{1}{2}BC\right)^2$

$\Rightarrow AB^2 = AD^2 + \frac{1}{4}BC^2$

$\Rightarrow AB^2 = AD^2 + \frac{AB^2}{4}$ [$\because$ BC=AB]

$\Rightarrow \frac{3}{4}AB^2 = AD^2$

$\Rightarrow 3AB^2 = 4AD^2$

17. Tick the correct answer and justify: In ΔABC, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm. the angles A and B are respectively:

(A) 90° **and** 30°

(B) 90° **and** 60°

(C) 30° **and** 90°

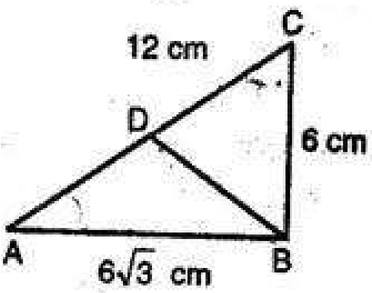
(D) 60° **and** 90°

Ans. (C) In ΔABC, we have, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm.

Now, $= \left(6\sqrt{3}\right)^2 + (6)^2 = 36\times3+36 = 108+36$ =144= $(AC)^2$

Thus, ΔABC is a right triangle, right angled at B.

$\therefore \angle$B = 90°



Let D be the midpoint of AC. We know that the midpoint of the hypotenuse of a right triangle is equidistant from the vertices.

AD=BD=CD

$\Rightarrow$ CD = BD = 6 cm [$\because$ CD = $\frac{1}{2}$AC]

Also, BC = 6 cm

$\therefore$ In ΔBDC, we have, BD = CD = BC

$\Rightarrow$ ΔBDC is equilateral

$\Rightarrow$ $\angle$ACB = 60°

$\therefore$ $\angle$A = $180^\circ - (\angle B + \angle C) = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$

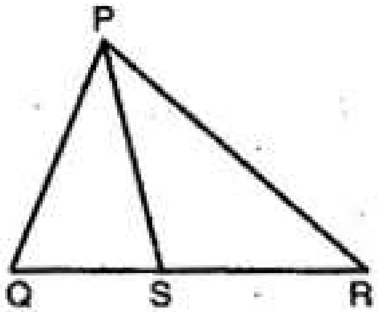
Thus, $\angle$A = 30° and $\angle$B = 90°

Chapter - 6

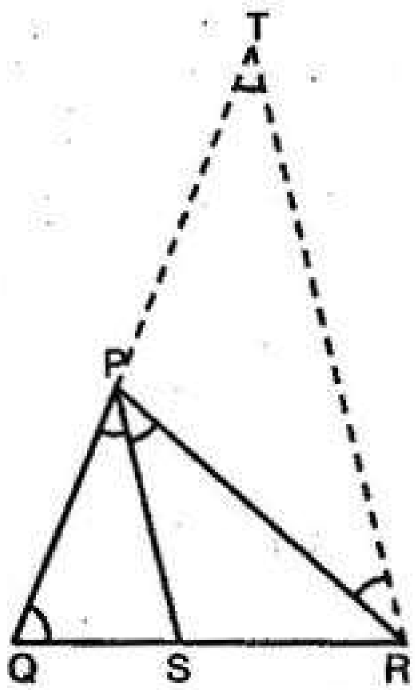
Triangles - Exercise 6.6

1. In the given figure, PS is the bisector of $\angle$ QPR of $\triangle$ PQR. Prove that

$$\frac{QS}{SR} = \frac{PQ}{PR}.$$



Ans. Given: PQR is a triangle and PS is the internal bisector of $\angle$ QPR meeting QR at S.



$\therefore \angle$ QPS = $\angle$ SPR

To prove: $\frac{QS}{SR} = \frac{PQ}{PR}$

Construction: Draw RT $\parallel$ SP to cut QP produced at T.

Proof: Since PS $\parallel$ TR and PR cuts them, hence,

$\angle$ SPR = $\angle$ PRT ……….(i) [Alternate $\angle$ s]

And $\angle$ QPS = $\angle$ PTR ……….(ii)[Corresponding $\angle$ s]

But $\angle$ QPS = $\angle$ SPR [Given]

$\therefore$ $\angle$ PRT = $\angle$ PTR[From eq. (i) & (ii)]

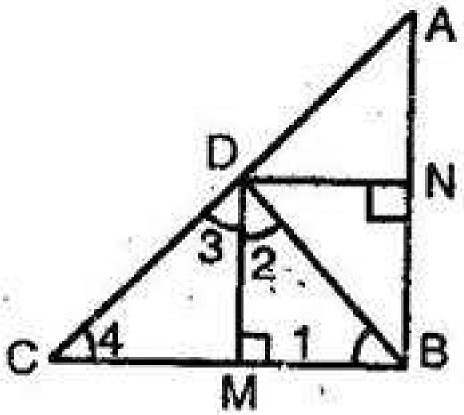
$\Rightarrow$ PT = PR……….(iii)

[Sides opposite to equal angles are equal]

Now, in ΔQRT,

RT $\parallel$ SP[By construction]

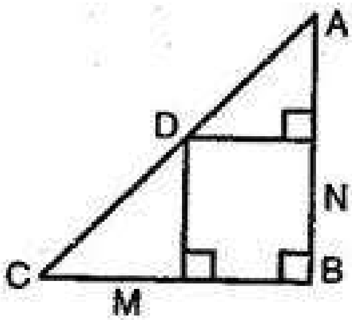
$\therefore \frac{QS}{SR} = \frac{PQ}{PT}$ [Thales theorem]



$\Rightarrow \frac{QS}{SR} = \frac{PQ}{PR}$ [From eq. (iii)]

2. In the given figure, D is a point on hypotenuse AC of ΔABC, BD $\perp$ AC, DM $\perp$ BC and

DN $\perp$ AB. Prove that:



(i) DM^2 = DN.MC

(ii) DN^2 = DM.AN

Ans. Since AB $\perp$ BC and DM $\perp$ BC

$\Rightarrow$ AB DM

Similarly, BC $\perp$ AB and DN $\perp$ AB

$\Rightarrow$ CB DN

$\therefore$ quadrilateral BMDN is a rectangle.

$\therefore$ BM=ND

= In ΔBMD, $\angle 1 + \angle BMD + \angle 2 =$

$180^\circ \Rightarrow \angle 1 + 90^\circ + \angle 2 = 180^\circ \Rightarrow \angle 1 +$

$\angle 2 = 90^\circ$

Similarly, in ΔDMC, $\angle 3 + \angle 4 = 90^\circ$

Since BD $\perp$ AC,

$\therefore \angle 2 + \angle 3 = 90^\circ$

Now, $\angle 1 + \angle 2 = 90^\circ$ and $\angle 2 + \angle 3 =$

$90^\circ \Rightarrow \angle 1 + \angle 2 = \angle 2 + \angle 3$

$\Rightarrow \angle 1 = \angle 3$

Also, $\angle 3 + \angle 4 = 90^\circ$ and $\angle 2 + \angle 3 = 90^\circ$

$\Rightarrow \angle 3 + \angle 4 = \angle 2 + \angle 3$

$\Rightarrow \angle 4 = \angle 2$

Thus, in ΔBMD and ΔDMC,

$\angle 1 = \angle 3$ and $\angle 4 = \angle 2$

$\therefore \Delta BMD \sim \Delta DMC$

$\Rightarrow \frac{BM}{DM} = \frac{MD}{MC}$

$\Rightarrow \frac{DN}{DM} = \frac{DM}{MC}$ [BM = ND]

$\Rightarrow DM^2$ = DN.MC

= Processing as in (i), we can prove that

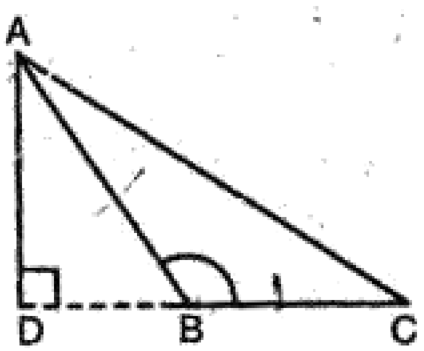
$\Delta BND \sim \Delta DNA$

$\Rightarrow \frac{BN}{DN} = \frac{ND}{NA}$

$\Rightarrow \frac{DM}{DN} = \frac{DN}{AN}$ [BN = DM]

$\Rightarrow DN^2$ = DM.AN

21 **In the given figure, ABC is a triangle in which $\angle$ ABC > 90° and AD $\perp$ CB produced. Prove that:**



$$AC^2 = AB^2 + BC^2 + 2BC.BD$$

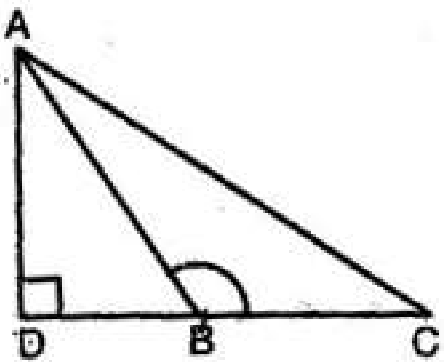
Ans. Given: ABC is a triangle in which $\angle$ ABC > 90° and AD $\perp$ CB produced.

To prove: $AC^2 = AB^2 + BC^2 + 2BC.BD$

Proof: Since Δ ADB is a right triangle, right angled at D, therefore, by Pythagoras theorem,

$AB^2 = AD^2 + DB^2$ ………(i)

Again, Δ ADB is a right triangle, right angled at D, therefore, by Pythagoras theorem,



$$AC^2 = AD^2 + DC^2$$

$$\Rightarrow AC^2 = AD^2 + (DB + BC)^2$$

$$\Rightarrow AC^2 = AD^2 + DB^2 + BC^2 + 2DB.BC$$

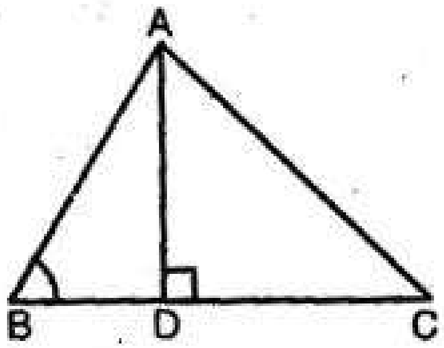
$$\Rightarrow AC^2 = \left(AD^2 + DB^2\right) + BC^2 + 2DB.BC$$

$$\Rightarrow AC^2 = AB^2 + BC^2 + 2DB.BC$$

[Using eq. (i)]

23 **In the given figure, ABC is a triangle in which $\angle$ ABC < 90° and AD $\perp$ BC produced. Prove that:**

$\mathbf{AC^2 = AB^2 + BC^2 - 2BC}$ **.BD**



Ans. Given: ABC is a triangle in which $\angle$ ABC < 90° and AD $\perp$ BC produced.

To prove: $AC^2 = AB^2 + BC^2 - 2BC$ .BD

Proof: Since Δ ADB is a right triangle, right angled at D, therefore, by Pythagoras theorem,

$AB^2 = AD^2 + BD^2$ ………(i)

Again, Δ ADB is a right triangle, right angled at D, therefore, by Pythagoras theorem,

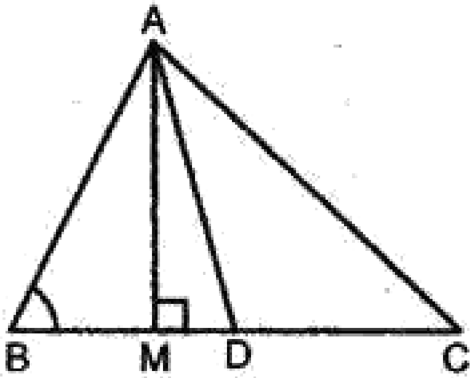
$AC^2 = AD^2 + DC^2$

$\Rightarrow AC^2 = AD^2 + (BC - BD)^2$

$\Rightarrow AC^2 = AD^2 + BC^2 + BD^2$ – 2BC.BD

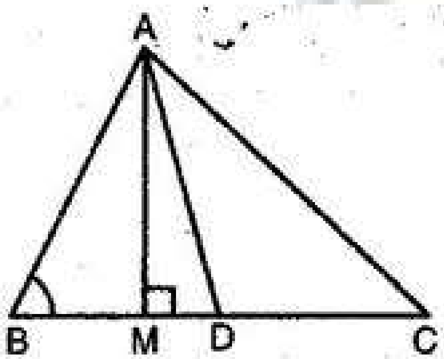
$\Rightarrow AC^2 = (AD^2 + DB^2) + BC^2$ – 2DB.BC

$\Rightarrow AC^2 = AB^2 + BC^2$ – 2DB.BC



[Using eq. (i)]

5. In the given figure, AD is a median of a triangle ABC and AM$\perp$ BC. Prove that:



(i) $\mathbf{AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2}$

(ii) $\mathbf{AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2}$

(iii) $\mathbf{AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC}$

Ans. Since $\angle$ AMD = 90°, therefore $\angle$ ADM < 90° and $\angle$ ADC > 90°

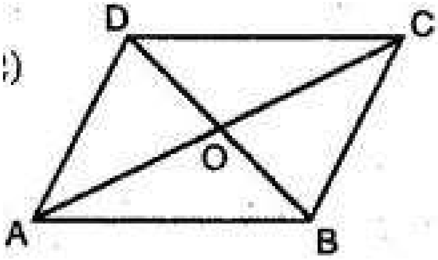
Thus, $\angle$ ADC is the acute angle and $\angle$ ADC is an obtuse angle.

(x) In ΔADC, $\angle$ ADC is an obtuse angle.

$\therefore AC^2 = AD^2 + DC^2 + 2DC.DM$

$$\Rightarrow AC^2 = AD^2 + \left(\frac{BC}{2}\right)^2 + 2.\frac{BC}{2}.DM$$

$$\Rightarrow AC^2 = AD^2 + \left(\frac{BC}{2}\right)^2 + BC.DM$$



$$\Rightarrow AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2 \text{……….(i)}$$

(vii) In ΔABD, $\angle$ ADM is an acute angle.

$$AB^2 = AD^2 + BD^2 - 2BD.DM$$

$$\Rightarrow AB^2 = AD^2 + \left(\frac{BC}{2}\right)^2 - 2.\frac{BC}{2}.DM$$

$$\Rightarrow AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2 \text{……….(ii)}$$

(g) From eq. (i) and eq. (ii),

$$AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$$

8. Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.

Ans. If AD is a median of ΔABC, then

$$AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2 \text{ [See Q.5 (iii)]}$$

Since the diagonals of a parallelogram bisect each other, therefore, BO and DO are medians of triangles ABC and ADC respectively.

$\therefore AB^2 + BC^2 = 2BO^2 + \frac{1}{2}AC^2$ ……….(i)

And $AD^2 + CD^2 = 2DO^2 + \frac{1}{2}AC^2$ ……….(ii)

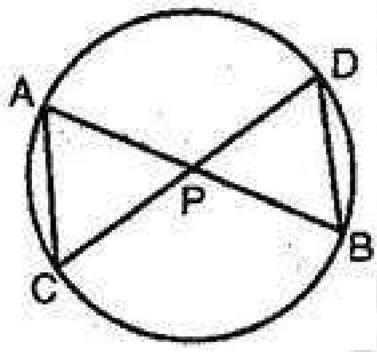
Adding eq. (i) and (ii),

$AB^2 + BC^2 + AD^2 + CD^2 = 2(BO^2 + DO^2) + AC^2$

$\Rightarrow AB^2 + BC^2 + AD^2 + CD^2 = 2\left(\frac{1}{4}BD^2 + \frac{1}{4}BD^2\right) + AC^2 \left[DO = \frac{1}{2}BD\right]$

$\Rightarrow AB^2 + BC^2 + AD^2 + CD^2 = AC^2 + BD^2$

13. In the given figure, two chords AB and CD intersect each other at the point P. Prove that:



(vii) ΔAPC ~ ΔDPB

(viii) AP.PB = CP.DP

Ans. (i) In the triangles APC and DPB,

$\angle$APC = $\angle$DPB [Vertically opposite angles]

$\angle$CAP = $\angle$BDP [Angles in same segment of a circle are equal]

$\therefore$ By AA-criterion of similarity,

Δ APC $\sim$ Δ DPB

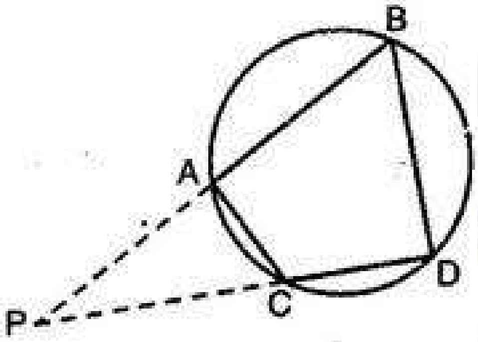
(ix) Since Δ APC $\sim$ Δ DPB

$\therefore \frac{AP}{DP} = \frac{CP}{PB} \Rightarrow$ AP x PB = CP x DP

(vi) In the give figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that:

10. Δ PAC $\sim$ Δ PDB

11. PA.PB = PC.PD



Ans. (i) In the triangles PAC and PDB,

$\angle$ APC = $\angle$ DPB [Common]

$\angle$ CAP = $\angle$ BDP [$\because$ $\angle$ BAC = $180^\circ - \angle$ PAC and $\angle$ PDB = $\angle$ CDB]

= $180^\circ - \angle$ BAC = $180^\circ - (180^\circ - \angle PAC)$ =

$\angle$ PAC] $\therefore$ By AA-criterion of similarity,

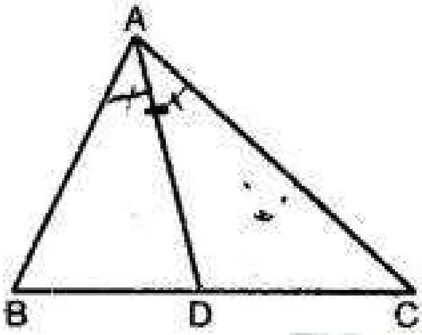
Δ APC $\sim$ Δ DPB

11. Since Δ APC $\sim$ Δ DPB

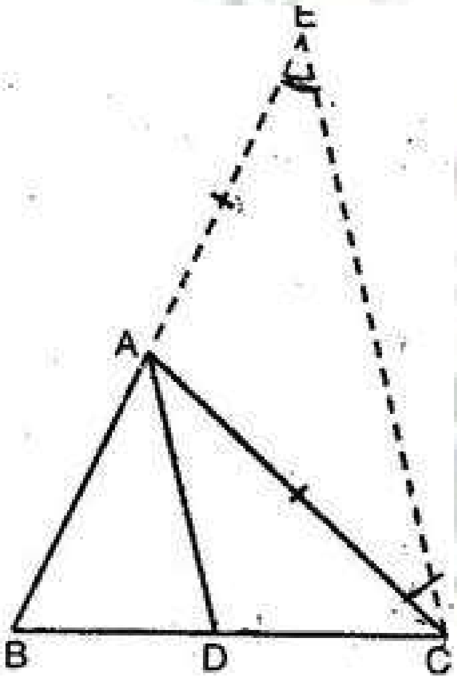
$\therefore \frac{AP}{DP} = \frac{CP}{PB}$

$\Rightarrow$ PA.PB = PC.PD

9. In the given figure, D is appointing on side BC of ΔABC such that $\frac{BD}{CD}=\frac{AB}{AC}$. Prove that AD is the bisector of $\angle$ BAC.



Ans. Given: ABC is a triangle and D is a point on BC such that $\frac{BD}{CD}=\frac{AB}{AC}$



To prove: AD is the internal bisector of $\angle$ BAC.

Construction: Produce BA to E such that AE = AC. Join CE.

Proof: In ΔAEC, since AE = AC

$\therefore \angle$ AEC = $\angle$ ACE ……….(i)

[Angles opposite to equal side of a triangle are equal]

Now, $\frac{BD}{CD} = \frac{AB}{AC}$ [Given]

$\Rightarrow \frac{BD}{CD} = \frac{AB}{AE}$ [$\because$ AE = AC, by construction]

$\therefore$ By converse of Basic Proportionality Theorem,

DA $\parallel$ CE

Now, since CA is a transversal,

$\therefore$ $\angle$BAD = $\angle$AEC ……….(ii) [Corresponding $\angle$s]

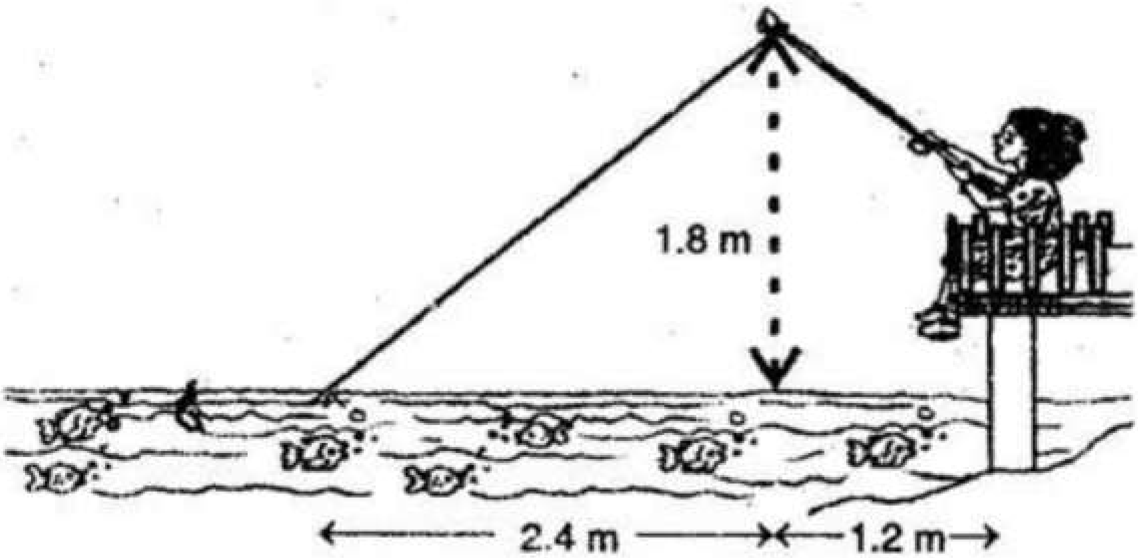
And $\angle$DAC = $\angle$ACE ……….(iii) [Alternate $\angle$s]

Also $\angle$AEC = $\angle$ACE [From eq. (i)]

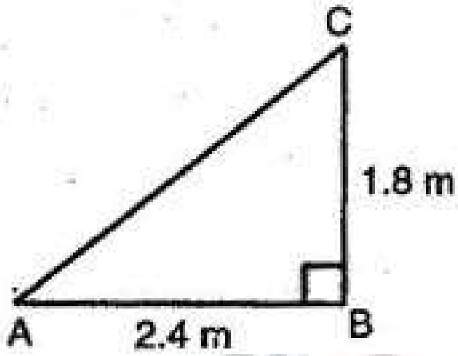
Hence, $\angle$BAD = $\angle$DAC [From eq. (ii) and (iii)]

Thus, AD bisects $\angle$BAC internally.

(iv) Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Fig.)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?



Ans. I. To find The length of AC.



By Pythagoras theorem,

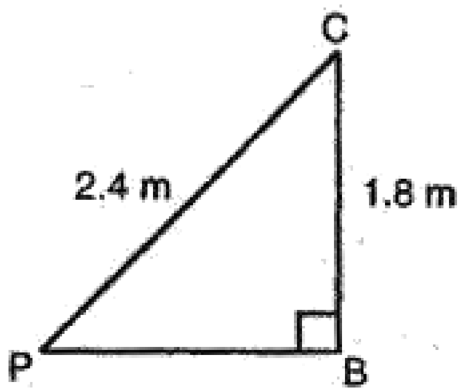
$AC^2 = (2.4)^2 + (1.8)^2$

$\Rightarrow AC^2 = 5.76 + 3.24 = 9.00$

$\Rightarrow$ AC = 3 m

$\therefore$ Length of string she has out= 3 m

Length of the string pulled at the rate of 5 cm/sec in 12 seconds



= (5 x 12) cm = 60 cm = 0.60 m

∴ Remaining string left out = 3 – 0.6 = 2.4 m

II. To find: The length of PB

$PB^2 = PC^2 - BC^2$

(iv) $(2.4)^2 - (1.8)^2$

(v) 5.76 – 3.24 = 2.52

$\Rightarrow$ PB = $\sqrt{2.52}$ = 1.59 (approx.)

Hence, the horizontal distance of the fly from Nazima after 12 seconds = 1.59 +

1.2 = 2.79 m (approx.)

PUNA INTERNATIONAL SCHOOL

- **CLASS – 10**
- **SUBJECT - MATHS**
- **CHAPTER - 10**

SAMPLE NOTE-BOOK

Chapter - 10

Circles –

Exercise 10.1

1. How many tangents can a circle have?

Ans. A circle can have infinitely many tangents since there are infinitely many points on the circumference of the circle and at each point of it, it has a unique tangent.

2. Fill in the blanks:

= **A tangent to a circle intersects it in _______________ point(s).**

= **A line intersecting a circle in two points is called a _______________.**

= **A circle can have _______________ parallel tangents at the most.**

= **The common point of a tangent to a circle and the circle is called _________.**

Ans. (i) A tangent to a circle intersects it in exactly one point.

= A line intersecting a circle in two points is called a secant.

= A circle can have two parallel tangents at the most.

= The common point of a tangent to a circle and the circle is called point of contact.

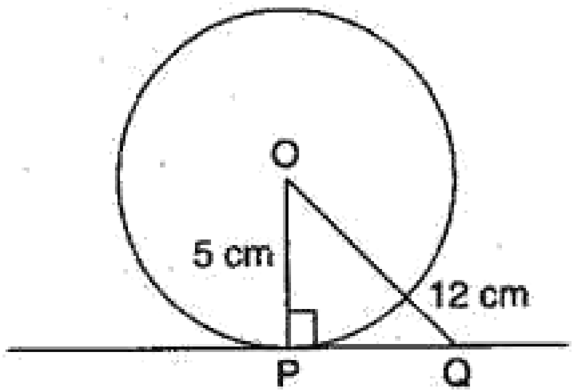
12 A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is:

(A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$ **cm**

Ans. (D) $\because$ PQ is the tangent and OP is the radius through the point of contact.

$\therefore \angle$ OPQ = 90° [The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\therefore$ In right triangle OPQ,



$OQ^2 = OP^2 + PQ^2$ [By Pythagoras theorem]

$\Rightarrow (12)^2 = (5)^2 + PQ^2$

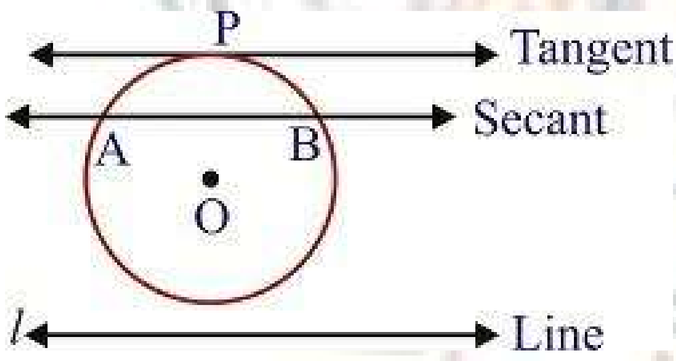
$\Rightarrow 144 = 25 + PQ^2$

$\Rightarrow PQ^2 = 144 - 25 = 119$

$\Rightarrow PQ = \sqrt{119}$ cm

18 Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Ans.



Chapter - 10

Circles - Exercise 10.2

In Q 1 to 3, choose the correct option and give justification.

= **From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is:**

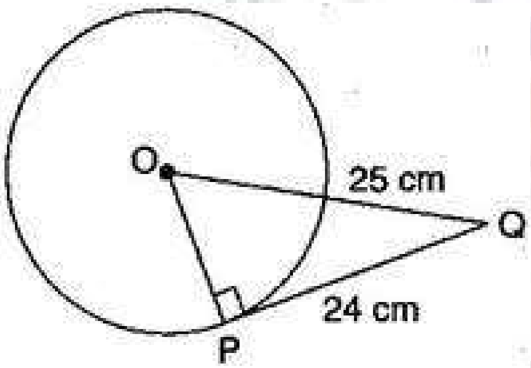
= **7 cm (B) 12 cm (C) 15 cm (D) 24.5 cm Ans.**

(A)

$\because \angle OPQ = 90^\circ$

[The tangent at any point of a circle is $\perp$ to the radius through

the point of contact]



$\therefore$ In right triangle OPQ,

$$OQ^2 = OP^2 + PQ^2$$

[By Pythagoras theorem]

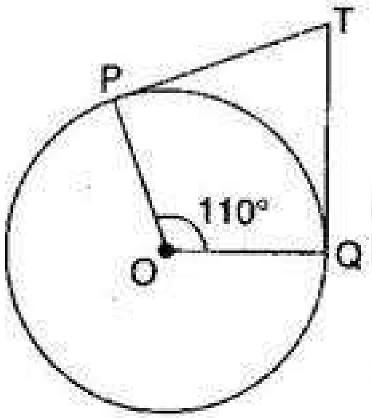
$$\Rightarrow (25)^2 = OP^2 + (24)^2$$

$$\Rightarrow 625 = OP^2 + 576$$

$\Rightarrow OP^2$ =625–576=49

$\Rightarrow$ OP = 7 cm

13 In figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle$ POQ = $110°$, then $\angle$ PTQ is equal to:



19 $60°$ **(B)** $70°$ **(C)** $80°$ **(D)** $90°$

Ans. (B)

$\angle$ POQ = $110°$, $\angle$ OPT = $90°$ and $\angle$ OQT = $90°$

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact] In

quadrilateral OPTQ,

$\angle$ POQ + $\angle$ OPT + $\angle$ OQT + $\angle$ PTQ = $360°$

[Angle sum property of quadrilateral]

$\Rightarrow 110° + 90° + 90°$ + $\angle$ PTQ= $360°$

$\Rightarrow 290°$ + $\angle$ PTQ= $360°$

$\Rightarrow$ $\angle$PTQ = 360° - 290°

$\Rightarrow$ $\angle$ PTQ = $70°$

(iv) If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then $\angle$ POA is equal to:

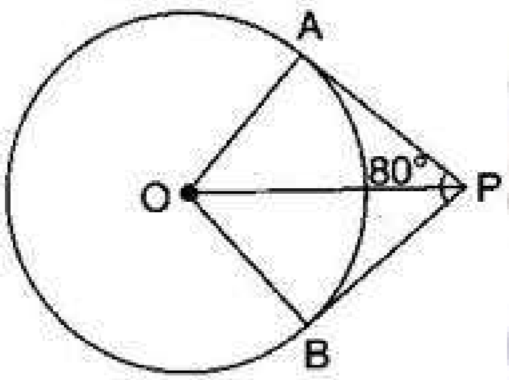
(ii) 50° **(B)** 60° **(C)** 70° **(D)** 80°

Ans. (A)

$\because \angle OAP = 90^\circ$

[The tangent at any point of a circle is $\perp$ to the radius through

the point of contact]



$\angle OPA = \frac{1}{2}\angle BPA = \frac{1}{2}\times 80^\circ = 40^\circ$

[Centre lies on the bisector of the

angle between the two tangents]

In OPA,

$\angle OAP + \angle OPA + \angle POA = 180^\circ$

[Angle sum property of a triangle]

$+ \angle$ POA=

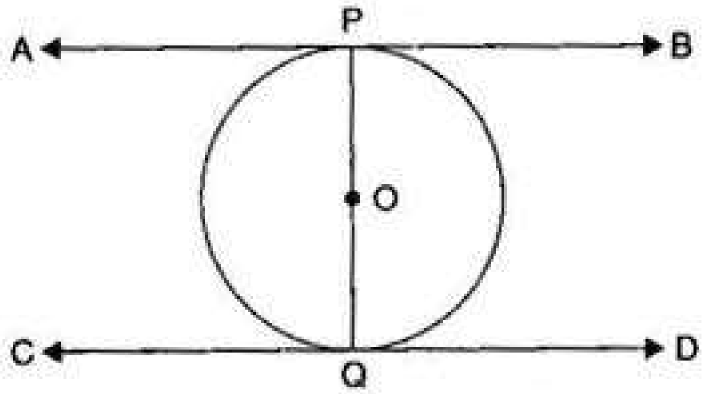
$+ \angle$ POA=

$\Rightarrow \angle POA = 50^\circ$

(a) Prove that the tangents drawn at the ends of a diameter of a circle are parallel. Ans.

Given: PQ is a diameter of a circle with centre O.

The lines AB and CD are the tangents at P and Q respectively.



To Prove: AB $\parallel$ CD

Proof: Since AB is a tangent to the circle at P and OP is the radius through the point of contact.

$\therefore \angle$ OPA = 90° ……….(i)

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact] $\because$ CD is a tangent to the circle at Q and OQ is the radius through the point of contact.

$\therefore \angle$ OQD = 90° ……….(ii)

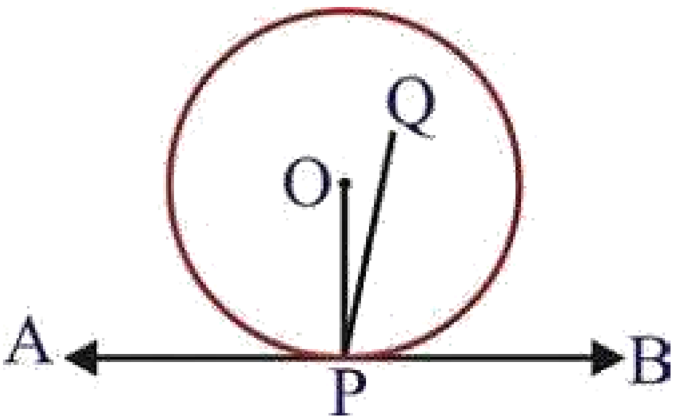
[The tangent at any point of a circle is $\perp$ to the radius through the point of contact] From eq. (i) and (ii), $\angle$ OPA = $\angle$ OQD

But these form a pair of equal alternate angles also, $\therefore$ AB ||

CD

5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Ans. Let AB be the tangent drawn at the point P on the circle with O.



If possible, let PQ be perpendicular to AB, not passing through O.

Join OP.

Since tangnet at a point to a circle is perpendicular to the radius through the point.

Therefore, AB $\perp$ OP $\Rightarrow$ $\angle$OPB = 90º

Also, $\angle$QPB = 90º [By construction]

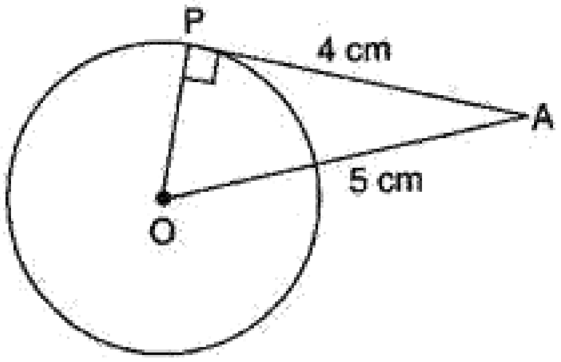
Therefore, $\angle$QPB = $\angle$OPB, which is not possible as a part cannot be equal to whole.

Thus, it contradicts our supposition.

Hence, the perpendicular at the point of contact to the tangent to a circle passes through the centre.

7. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Ans. We know that the tangent at any point of a circle is $\perp$ to the radius through the point of contact.



$\therefore \angle$ OPA = 90°

$\therefore\ OA^2 = OP^2 + AP^2$

[By Pythagoras theorem]

$\Rightarrow (5)^2 = (\mathrm{OP})^2 + (4)^2$

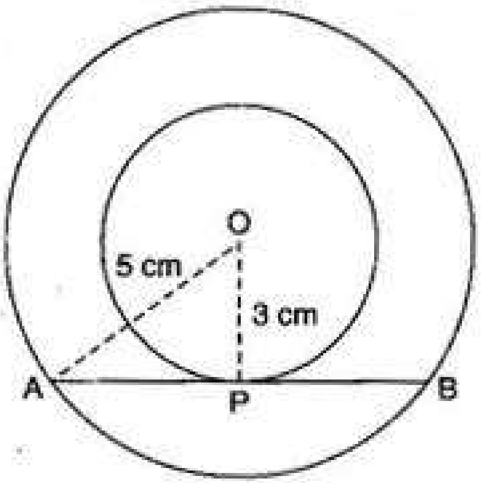
$\Rightarrow\ 25 = (\mathrm{OP})^2 + 16$

$\Rightarrow OP^2$ = 9

$\Rightarrow$ OP = 3 cm

(iii) Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Ans. Let O be the common centre of the two concentric circles.



Let AB be a chord of the larger circle which touches the smaller circle at P.

Join OP and OA.

Then, $\angle$ OPA = 90°

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\therefore$ $OA^2 = OP^2 + AP^2$

[By Pythagoras theorem]

$\Rightarrow (5)^2 = (3)^2 + AP^2$

$\Rightarrow 25 = 9 + AP^2$

$\Rightarrow AP^2 = 16$

$\Rightarrow$ AP = 4 cm

Since the perpendicular from the centre of a circle to a chord bisects the chord, therefore

AP = BP = 4 cm

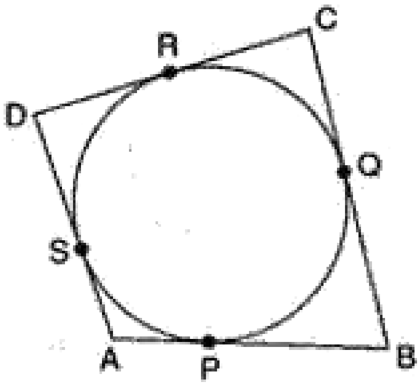
$\Rightarrow$ AB=AP+BP

(ii) AP+AP=2AP

(iii) 2×4 = 8 cm

(iv) A quadrilateral ABCD is drawn to circumscribe a circle (see figure). Prove that:

AB+CD=AD+BC



Ans. We know that the tangents from an external point to a circle are equal. $\therefore$ AP = AS

……….(i)

BP = BQ ……….(ii)

CR = CQ ……….(iii)

DR = DS……….(iv)

On adding eq. (i), (ii), (iii) and (iv), we get

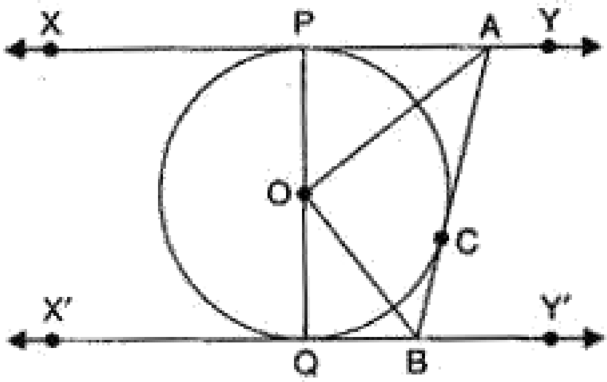
(AP + BP) + (CR + DR)

= (AS + BQ) + (CQ + DS)

$\Rightarrow$ AB+CD=(AS+DS)+(BQ+CQ)

$\Rightarrow$ AB+CD=AD+BC

8. In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle$ AOB = 90°.

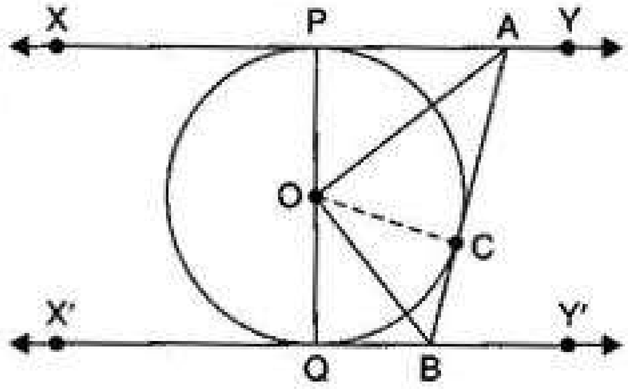


Ans. Given: In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B.

To Prove: $\angle$ AOB = 90°

Construction: Join OC

Proof: $\angle$ OPA = 90° ……….(i)



$\angle$ OCA = 90° ……….(ii)

[Tangent at any point of a circle is $\perp$ to

the radius through the point of contact]

In right angled triangles OPA and OCA,

$\angle$ OPA = $\angle$ OCA = 90º

OA = OA [Common]

AP = AC [Tangents from an external

point to a circle are equal]

$\therefore \Delta OPA \cong \Delta OCA$

[RHS congruence criterion]

$\therefore \angle OAP = \angle OAC$ [By C.P.C.T.]

$\Rightarrow \angle OAC = \frac{1}{2}\angle PAB$ ……….(iii)

Similarly, $\angle OBQ = \angle OBC$

$\Rightarrow \angle OBC = \frac{1}{2}\angle QBA$ ……….(iv)

$\because$ XY $\parallel$ X'Y' and a transversal AB intersects them.

$\therefore \angle PAB + \angle QBA = 180^\circ$

[Sum of the consecutive interior angles on the same side of the transversal is 180°]

$\Rightarrow \frac{1}{2}\angle PAB + \frac{1}{2}\angle QBA$

$= \frac{1}{2}\times 180^\circ$ ……….(v)

$\Rightarrow \angle OAC + \angle OBC = 90^\circ$

[From eq. (iii) & (iv)]

In ΔAOB,

$\angle OAC + \angle OBC + \angle AOB = 180^\circ$

[Angel sum property of a triangle]

$+ \angle AOB =$ [From eq. (v)]

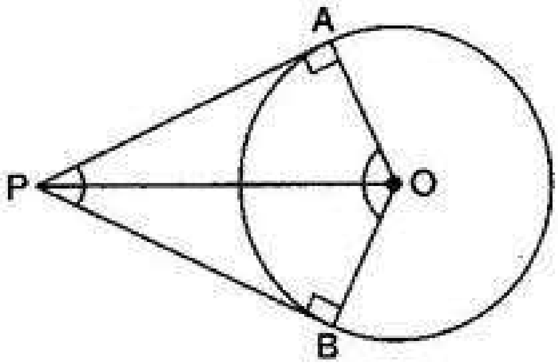
$\Rightarrow \angle AOB =$

Hence proved.

(i) Prove that the angel between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Ans. $\angle$ OAP = 90° ……….(i)

$\angle$ OBP = 90° ……….(ii)



[Tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\because$ OAPB is quadrilateral.

$\therefore \angle$ APB + $\angle$ AOB + $\angle$ OAP + $\angle$ OBP = 360°
[Angle sum property of a quadrilateral]

$\Rightarrow \angle$ APB+ $\angle$ AOB+ $90^\circ + 90^\circ = 360^\circ$

[From eq. (i) & (ii)]

$\Rightarrow \angle$ APB + $\angle$ AOB = 180°

$\therefore \angle$ APB and $\angle$ AOB are supplementary.

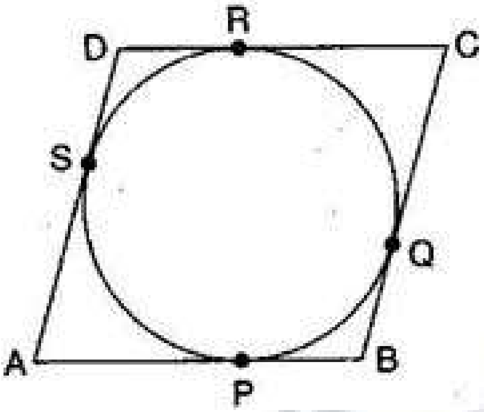
(ii) Prove that the parallelogram circumscribing a circle is a rhombus. Ans. Given:

ABCD is a parallelogram circumscribing a circle.

To Prove: ABCD is a rhombus.

Proof: Since, the tangents from an external point to a circle are equal.

$\therefore$ AP = AS ……….(i)



BP = BQ ……….(ii)

CR = CQ ……….(iii)

DR = DS……….(iv)

On adding eq. (i), (ii), (iii) and (iv), we get

(AP + BP) + (CR + DR)

= (AS + BQ) + (CQ + DS)

$\Rightarrow$ AB+CD=(AS+DS)+(BQ+CQ)

$\Rightarrow$ AB+CD=AD+BC

$\Rightarrow$ AB+AB=AD+AD

[Opposite sides of $\parallel$gm are equal]

$\Rightarrow$ 2AB = 2AD

$\Rightarrow$ AB=AD

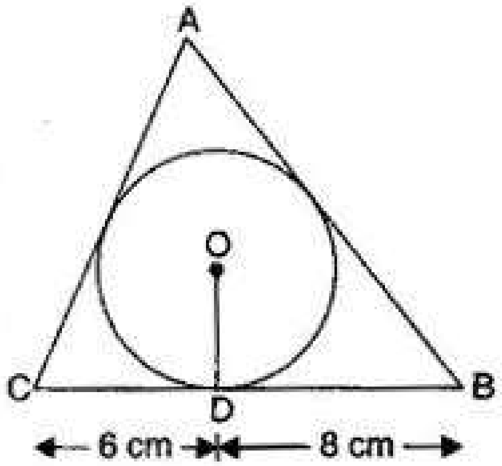
But AB = CD and AD = BC

[Opposite sides of || gm]

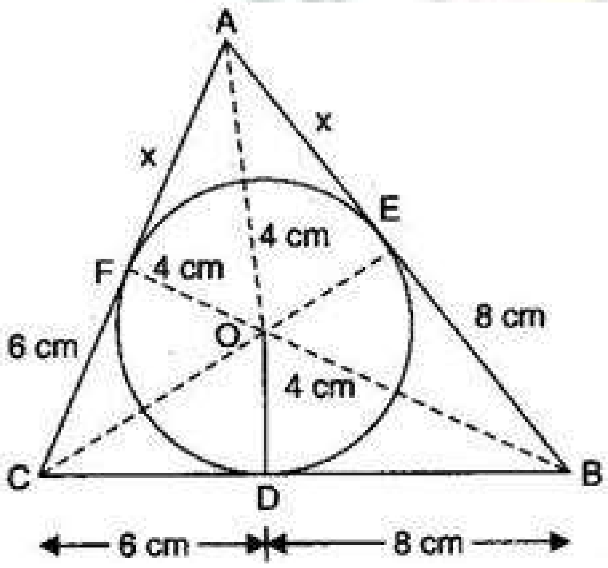
∴ AB=BC=CD=AD

∴ Parallelogram ABCD is a rhombus.

10. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see figure). Find the sides AB and AC.



Ans. Join OE and OF. Also join OA, OB and OC.



Since BD = 8 cm

∴ BE = 8 cm

[Tangents from an external point to a circle are equal]

Since CD = 6 cm

$\therefore$ CF = 6 cm

[Tangents from an external point to a circle are equal] Let AE =

AF = x

Since OD = OE = OF = 4 cm

[Radii of a circle are equal]

$\therefore$ Semi-perimeter of ΔABC = $\dfrac{(x+6)+(x+8)+(6+8)}{2} = (x+14)$ cm

$\therefore$ Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(x+14)(x+14-14)(x+14-\overline{x+8})(x+14-\overline{x+6})}$

$= \sqrt{(x+14)(x)(6)(8)}$ cm^2

Now, Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$

$= \dfrac{(6+8)4}{2} + \dfrac{(x+6)4}{2} + \dfrac{(x+8)4}{2}$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$

$= 28 + 2x + 12 + 2x + 16$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)} = 4x + 56$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)} = 4(x+14)$

Squaring both sides,

$$(x+14)(x)(6)(8)=16(x+14)^2$$

$$\Rightarrow 3x = x+14$$

$$\Rightarrow 2x = 14$$

$$\Rightarrow x = 7$$

$\therefore$ AB = $x+8$ = 7 + 8 = 15 cm

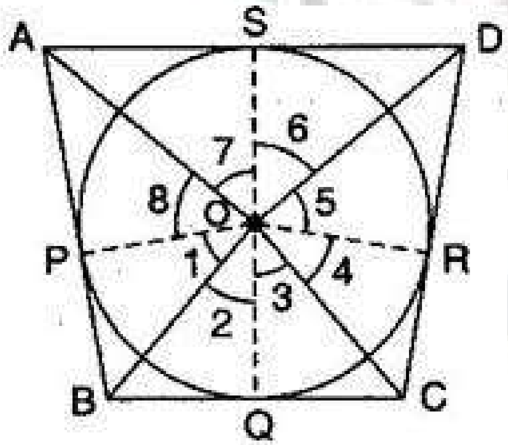
And AC = $x+6$ = 7 + 6 = 13 cm

(ii) Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Ans. Given: ABCD is a quadrilateral circumscribing a circle whose centre is O.

To prove: (i) $\angle$ AOB + $\angle$ COD = 180° (ii) $\angle$ BOC + $\angle$ AOD = 180°

Construction: Join OP, OQ, OR and OS.



Proof: Since tangents from an external point to a circle are equal.

$\therefore$ AP = AS,

BP = BQ ……….(i)

CQ=CR

DR=DS

In ΔOBP and ΔOBQ,

OP = OQ [Radii of the same circle]

OB = OB [Common]

BP = BQ [From eq. (i)]

$\therefore \Delta$OPB $\cong \Delta$ OBQ [By SSS congruence criterion]

$\therefore \angle 1 = \angle 2$ [By C.P.C.T.]

Similarly, $\angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$

Since, the sum of all the angles round a point is equal to $360°$.

$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$\Rightarrow \angle 1 + \angle 1 + \angle 4 + \angle 4 + \angle 5 + \angle 5 + \angle 8 + \angle 8 = 360°$

$\Rightarrow 2(\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360°$

$\Rightarrow \angle 1 + \angle 4 + \angle 5 + \angle 8 = 180°$

$\Rightarrow (\angle 1 + \angle 5) + (\angle 4 + \angle 8) = 180°$

$\Rightarrow \angle$ AOB + $\angle$ COD = $180°$

Similarly, we can prove that

$\angle$ BOC + $\angle$ AOD = $180°$

Chapter - 10

Circles - Exercise 10.1

1. How many tangents can a circle have?

Ans. A circle can have infinitely many tangents since there are infinitely many points on the circumference of the circle and at each point of it, it has a unique tangent.

2. Fill in the blanks:

= **A tangent to a circle intersects it in _______________ point(s).**

= **A line intersecting a circle in two points is called a _______________.**

= **A circle can have _______________ parallel tangents at the most.**

= **The common point of a tangent to a circle and the circle is called _________.**

Ans. (i) A tangent to a circle intersects it in <u>exactly one</u> point.

= A line intersecting a circle in two points is called a <u>secant</u>.

= A circle can have <u>two</u> parallel tangents at the most.

= The common point of a tangent to a circle and the circle is called <u>point of contact</u>.

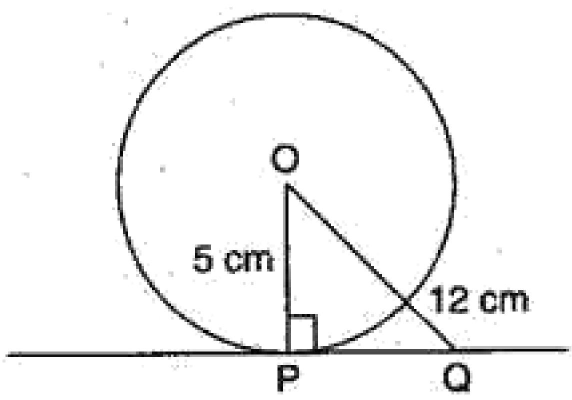
14 A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is:

(A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$ **cm**

Ans. (D) $\because$ PQ is the tangent and OP is the radius through the point of contact.

$\therefore \angle$ OPQ = 90° [The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\therefore$ In right triangle OPQ,



$OQ^2 = OP^2 + PQ^2$ [By Pythagoras theorem]

$\Rightarrow (12)^2 = (5)^2 + PQ^2$

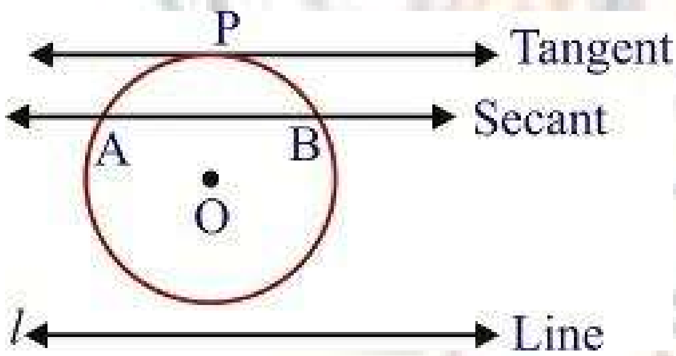
$\Rightarrow 144 = 25 + PQ^2$

$\Rightarrow PQ^2 = 144 - 25 = 119$

$\Rightarrow PQ = \sqrt{119}$ cm

20 Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Ans.



Chapter - 10

Circles - Exercise 10.2

In Q 1 to 3, choose the correct option and give justification.

= **From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is:**

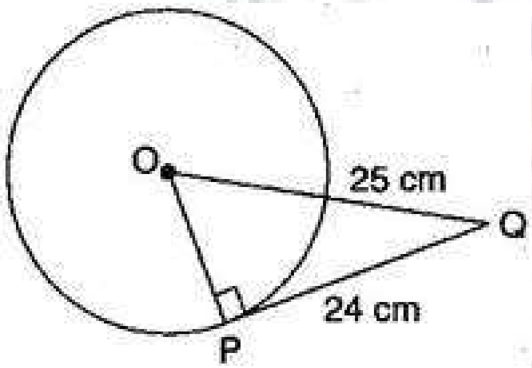
= **7 cm (B) 12 cm (C) 15 cm (D) 24.5 cm Ans.**

(A)

$\because \angle$ OPQ = 90°

[The tangent at any point of a circle is $\perp$ to the radius through

the point of contact]



$\therefore$ In right triangle OPQ,

$OQ^2 = OP^2 + PQ^2$

[By Pythagoras theorem]

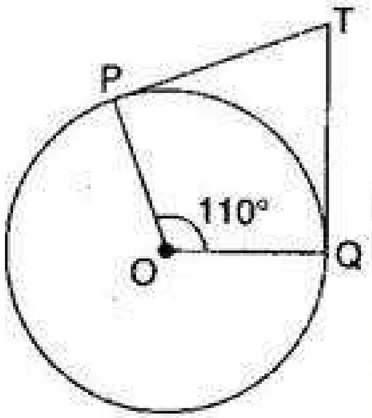
$\Rightarrow (25)^2 = OP^2 + (24)^2$

$\Rightarrow 625 = OP^2 + 576$

$\Rightarrow OP^2$ =625–576=49

$\Rightarrow$ OP = 7 cm

15 In figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle$ POQ = $110°$, then $\angle$ PTQ is equal to:



(A) $60°$ **(B)** $70°$ **(C)** $80°$ **(D)** $90°$

Ans. (B)

$\angle$ POQ = $110°$, $\angle$ OPT = $90°$ and $\angle$ OQT = $90°$

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact] In quadrilateral OPTQ,

$\angle$ POQ + $\angle$ OPT + $\angle$ OQT + $\angle$ PTQ = $360°$

[Angle sum property of quadrilateral]

$\Rightarrow 110° + 90° + 90°$ + $\angle$ PTQ= $360°$

$\Rightarrow 290°$ + $\angle$ PTQ= $360°$

$\Rightarrow$ $\angle$PTQ = 360° - 290°

$\Rightarrow \angle$ PTQ = $70°$

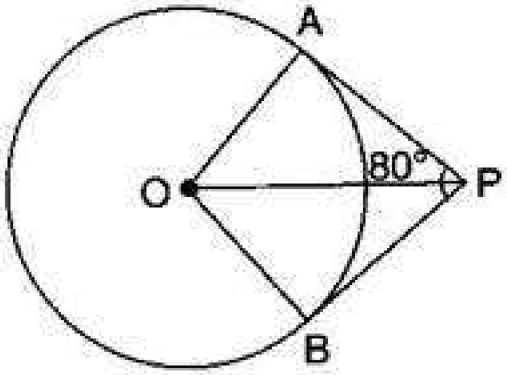
16. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then $\angle$ POA is equal to:

(A) 50° **(B)** 60° **(C)** 70° **(D)** 80°

Ans. (A)

$\because \angle$ OAP = 90°

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]



$\angle$ OPA = $\frac{1}{2}\angle$ BPA = $\frac{1}{2}\times 80^\circ = 40^\circ$

[Centre lies on the bisector of the angle between the two tangents]

In OPA,

$\angle$ OAP + $\angle$ OPA + $\angle$ POA = 180°

[Angle sum property of a triangle]

+ $\angle$ POA=

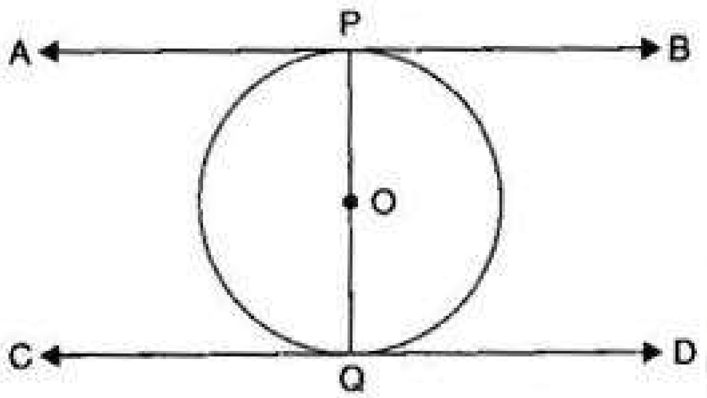
+ $\angle$ POA=

$\Rightarrow \angle$ POA = 50°

(b) Prove that the tangents drawn at the ends of a diameter of a circle are parallel. Ans.

Given: PQ is a diameter of a circle with centre O.

The lines AB and CD are the tangents at P and Q respectively.



To Prove: AB $\parallel$ CD

Proof: Since AB is a tangent to the circle at P and OP is the radius through the point of contact.

$\therefore \angle$ OPA = 90° ……….(i)

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact] $\because$ CD is a tangent to the circle at Q and OQ is the radius through the point of contact.

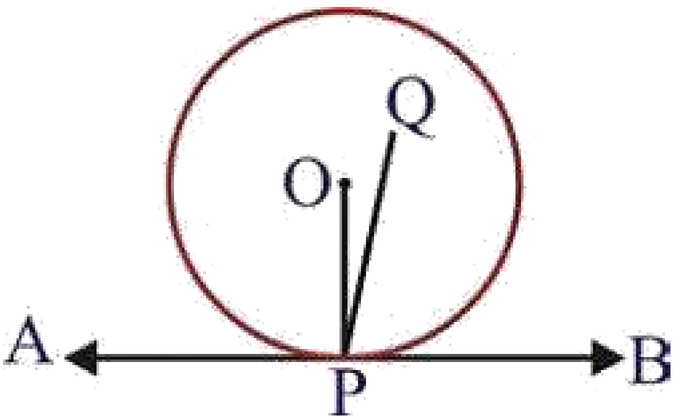
$\therefore \angle$ OQD = 90° ……….(ii)

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact] From eq. (i) and (ii), $\angle$ OPA = $\angle$ OQD

But these form a pair of equal alternate angles also, $\therefore$ AB || CD

6. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Ans. Let AB be the tangent drawn at the point P on the circle with O.



If possible, let PQ be perpendicular to AB, not passing through O.

Join OP.

Since tangnet at a point to a circle is perpendicular to the radius through the point.

Therefore, AB $\perp$ OP $\Rightarrow$ $\angle$OPB = 90º

Also, $\angle$QPB = 90º [By construction]

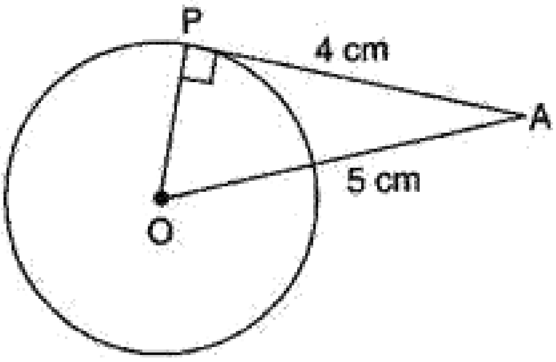
Therefore, $\angle$QPB = $\angle$OPB, which is not possible as a part cannot be equal to whole.

Thus, it contradicts our supposition.

Hence, the perpendicular at the point of contact to the tangent to a circle passes through the centre.

8. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Ans. We know that the tangent at any point of a circle is $\perp$ to the radius through the point of contact.



$\therefore \angle$ OPA = 90°

$\therefore \; OA^2 = \; OP^2 + AP^2$

[By Pythagoras theorem]

$\Rightarrow (5)^2 = (\text{OP})^2 + (4)^2$

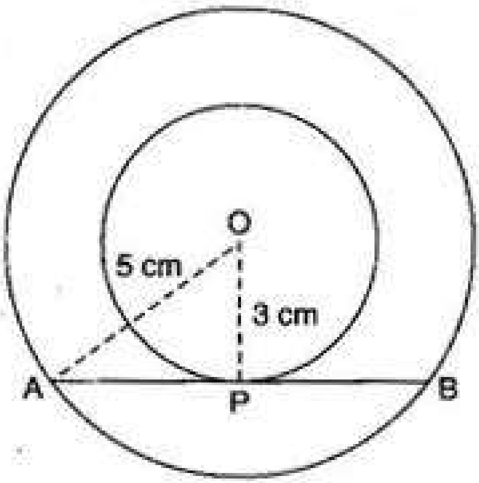
$\Rightarrow \; 25 = (\text{OP})^2 + 16$

$\Rightarrow OP^2 = 9$

$\Rightarrow$ OP = 3 cm

(iv) Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Ans. Let O be the common centre of the two concentric circles.



Let AB be a chord of the larger circle which touches the smaller circle at P.

Join OP and OA.

Then, $\angle$ OPA = 90°

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\therefore$ $OA^2 = OP^2 + AP^2$

[By Pythagoras theorem]

$\Rightarrow (5)^2 = (3)^2 + AP^2$

$\Rightarrow 25 = 9 + AP^2$

$\Rightarrow AP^2 = 16$

$\Rightarrow$ AP = 4 cm

Since the perpendicular from the centre of a circle to a chord bisects the chord, therefore

AP = BP = 4 cm

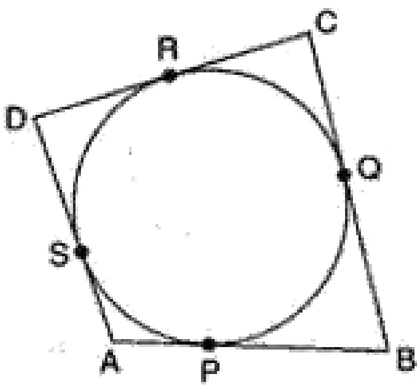
$\Rightarrow$ AB=AP+BP

(iv) AP+AP=2AP

(v) 2×4 = 8 cm

(v) A quadrilateral ABCD is drawn to circumscribe a circle (see figure). Prove that:

AB+CD=AD+BC



Ans. We know that the tangents from an external point to a circle are equal. $\therefore$ AP = AS

……….(i)

BP = BQ ……….(ii)

CR = CQ ……….(iii)

DR = DS……….(iv)

On adding eq. (i), (ii), (iii) and (iv), we get

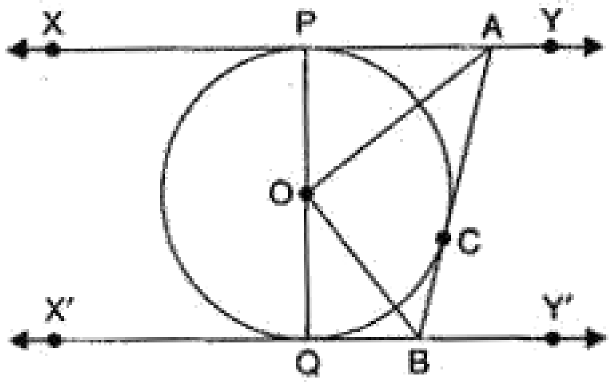
(AP + BP) + (CR + DR)

= (AS + BQ) + (CQ + DS)

$\Rightarrow$ AB+CD=(AS+DS)+(BQ+CQ)

$\Rightarrow$ AB+CD=AD+BC

9. In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle$ AOB = 90°.

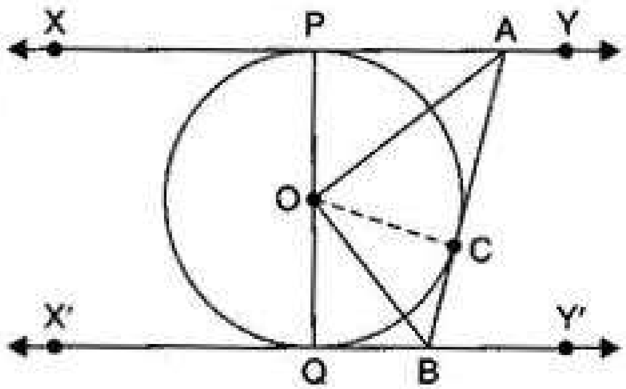


Ans. Given: In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B.

To Prove: $\angle$ AOB = 90°

Construction: Join OC

Proof: $\angle$ OPA = 90° ……….(i)



$\angle$ OCA = 90° ……….(ii)

[Tangent at any point of a circle is $\perp$ to

the radius through the point of contact]

In right angled triangles OPA and OCA,

$\angle$ OPA = $\angle$ OCA = 90º

OA = OA [Common]

AP = AC [Tangents from an external

point to a circle are equal]

$\therefore \Delta_{OPA} \cong \Delta_{OCA}$

[RHS congruence criterion]

$\therefore \angle$ OAP = $\angle$ OAC [By C.P.C.T.]

$\Rightarrow \angle$ OAC = $\frac{1}{2}\angle$PAB ……….(iii)

Similarly, $\angle$ OBQ = $\angle$ OBC

$\Rightarrow \angle$ OBC = $\frac{1}{2}\angle$QBA ……….(iv)

$\because$ XY $\parallel$ X'Y' and a transversal AB intersects them.

$\therefore \angle$ PAB + $\angle$ QBA = 180°

[Sum of the consecutive interior angles on the same side of the transversal is 180°]

$\Rightarrow \frac{1}{2}\angle$PAB + $\frac{1}{2}\angle$QBA

9. $\frac{1}{2}\times 180^\circ$……….(v)

$\Rightarrow \angle$ OAC + $\angle$ OBC = 90°

[From eq. (iii) & (iv)]

In ΔAOB,

$\angle$ OAC + $\angle$ OBC + $\angle$ AOB = 180°

[Angel sum property of a triangle]

+ $\angle$ AOB = [From eq. (v)]

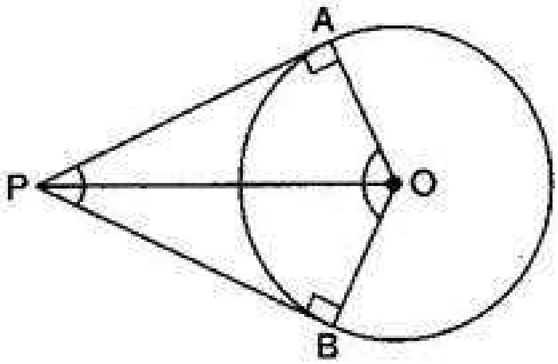
$\Rightarrow \angle$ AOB =

Hence proved.

(ii) Prove that the angel between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Ans. $\angle$ OAP = 90° ……….(i)

$\angle$ OBP = 90° ……….(ii)



[Tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\because$ OAPB is quadrilateral.

$\therefore$ $\angle$ APB + $\angle$ AOB + $\angle$ OAP + $\angle$ OBP = 360°

[Angle sum property of a quadrilateral]

$\Rightarrow$ $\angle$ APB+ $\angle$ AOB+ 90° + 90° = 360°

[From eq. (i) & (ii)]

$\Rightarrow$ $\angle$ APB + $\angle$ AOB = 180°

$\therefore$ $\angle$ APB and $\angle$ AOB are supplementary.

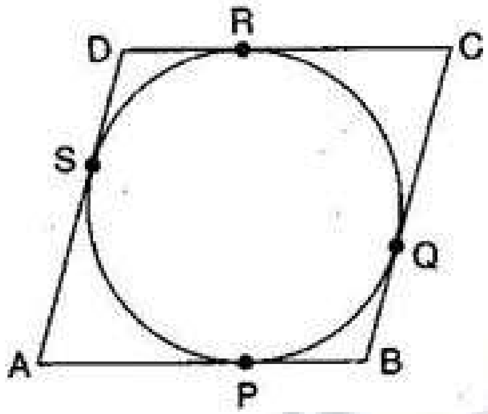
(iii) Prove that the parallelogram circumscribing a circle is a rhombus. Ans. Given:

ABCD is a parallelogram circumscribing a circle.

To Prove: ABCD is a rhombus.

Proof: Since, the tangents from an external point to a circle are equal.

$\therefore$ AP = AS ……….(i)



BP = BQ ……….(ii)

CR = CQ ……….(iii)

DR = DS……….(iv)

On adding eq. (i), (ii), (iii) and (iv), we get

(AP + BP) + (CR + DR)

= (AS + BQ) + (CQ + DS)

$\Rightarrow$ AB+CD=(AS+DS)+(BQ+CQ)

$\Rightarrow$ AB+CD=AD+BC

$\Rightarrow$ AB+AB=AD+AD

[Opposite sides of $\parallel$ gm are equal]

$\Rightarrow$ 2AB = 2AD

$\Rightarrow$ AB=AD

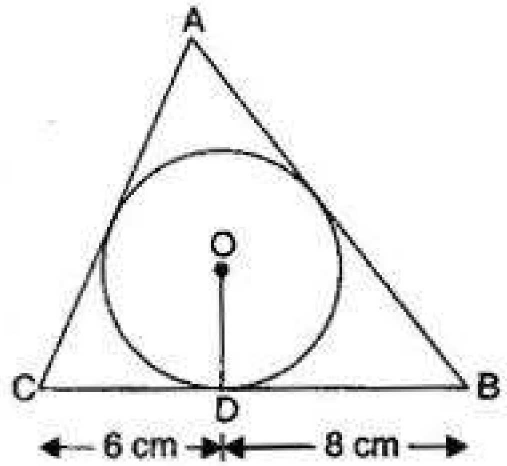
But AB = CD and AD = BC

[Opposite sides of || gm]

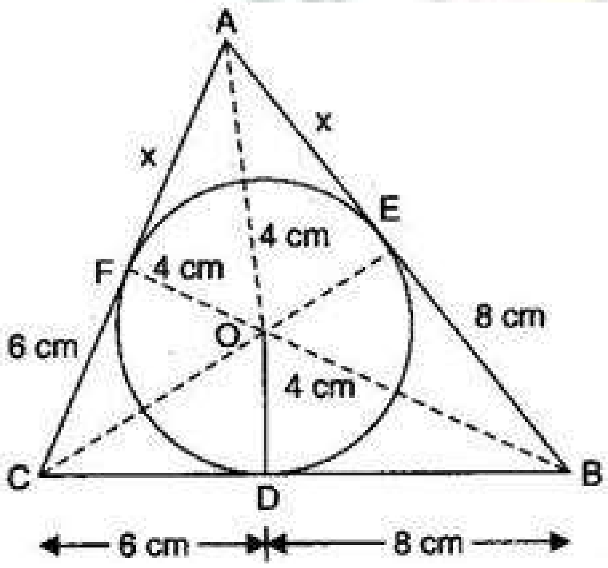
∴ AB=BC=CD=AD

∴ Parallelogram ABCD is a rhombus.

11. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see figure). Find the sides AB and AC.



Ans. Join OE and OF. Also join OA, OB and OC.



Since BD = 8 cm

∴ BE = 8 cm

[Tangents from an external point to a circle are equal]

Since CD = 6 cm

$\therefore$ CF = 6 cm

[Tangents from an external point to a circle are equal] Let AE =

AF = x

Since OD = OE = OF = 4 cm

[Radii of a circle are equal]

$\therefore$ Semi-perimeter of ΔABC = $\frac{(x+6)+(x+8)+(6+8)}{2} = (x+14)$ cm

$\therefore$ Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{(x+14)(x+14-14)(x+14-\overline{x+8})(x+14-\overline{x+6})}$

= $\sqrt{(x+14)(x)(6)(8)}$ cm^2

Now, Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$

= $\frac{(6+8)4}{2} + \frac{(x+6)4}{2} + \frac{(x+8)4}{2}$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$

= $28+2x+12+2x+16$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$ = $4x+56$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$ = $4(x+14)$

Squaring both sides,

$$(x+14)(x)(6)(8)=16(x+14)^2$$

$$\Rightarrow 3x = x+14$$

$$\Rightarrow 2x = 14$$

$$\Rightarrow x = 7$$

∴ AB = $x+8$ = 7 + 8 = 15 cm

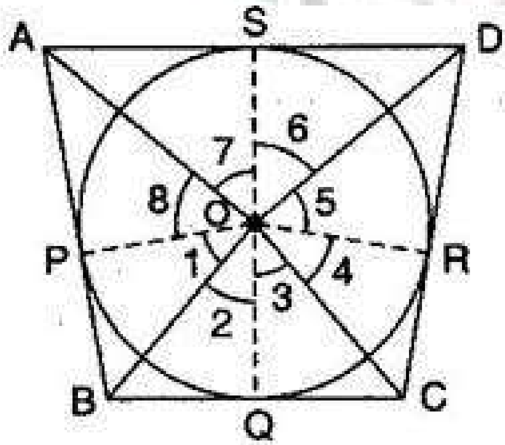
And AC = $x+6$ = 7 + 6 = 13 cm

(iii) Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Ans. Given: ABCD is a quadrilateral circumscribing a circle whose centre is O.

To prove: (i) $\angle$ AOB + $\angle$ COD = 180° (ii) $\angle$ BOC + $\angle$ AOD = 180°

Construction: Join OP, OQ, OR and OS.



Proof: Since tangents from an external point to a circle are equal.

∴ AP = AS,

BP = BQ ……….(i)

CQ=CR

DR=DS

In ΔOBP and ΔOBQ,

OP = OQ [Radii of the same circle]

OB = OB [Common]

BP = BQ [From eq. (i)]

$\therefore \Delta$OPB $\cong \Delta$ OBQ [By SSS congruence criterion]

$\therefore \angle 1 = \angle 2$ [By C.P.C.T.]

Similarly, $\angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$

Since, the sum of all the angles round a point is equal to $360°$.

$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$\Rightarrow \angle 1 + \angle 1 + \angle 4 + \angle 4 + \angle 5 + \angle 5 + \angle 8 + \angle 8 = 360°$

$\Rightarrow 2(\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360°$

$\Rightarrow \angle 1 + \angle 4 + \angle 5 + \angle 8 = 180°$

$\Rightarrow (\angle 1 + \angle 5) + (\angle 4 + \angle 8) = 180°$

$\Rightarrow \angle$ AOB + $\angle$ COD = $180°$

Similarly, we can prove that

$\angle$ BOC + $\angle$ AOD = $180°$